Magnetic instability and slow-wave propagation in a rotating fluid sphere

*Ataru Sakuraba[1]

1. Department of Earth and Planetary Science, University of Tokyo

We investigate instability of a toroidal basic field cofined in a rotating, finitely conducting and inviscid fluid sphere. In order to represent a system where the Lorentz-force term dominates in the vorticity equation like in the Earth's core, we use the magnetostrophic approximation by which the inertial term is exactly zero. The equation is linearized and represented in the azimuthal wavenumber (m) space. The remaining meridional space is discretized using the finite difference method. In this method, the grid points do not necessarily fall on the spherical surface (the core-mantle boundary), but the second-order accuracy can be kept by carefully implementing the boundary condition. The numerical method is verified by comparing the exact solution of the magnetic decay mode and our previous numerical code. The Arnoldi method and the inverse power method are used to solve the eigenvalue problem about the growth rate of the perturbed magnetic and velocity fields.

We generally assumed a basic toroidal field that is proportional to $(1 - s^2 - z^2)\ s^k$ (k=1, 3, 5, ...) or $(1 - s^2 - z^2)\ z\ s^{k-1}$ (k = 2, 4, 6, ...) where (s, z) are the cylindrical coordinates. This basic field is exactly zero at the spherical surface and the position of the intensity maximum tends to approach the equator on the spherical surface as k increases. When the Elsasser number, the only dimensionless parameter representing the square of the basic field intensity, increases and exceeds a certain value, the system turns to be unstable and a slow magnetostrophic wave propagates eastward (prograde) or westward (retrograde). The critical Elsasser number is not significantly dependent on k; the critical value is basically O(1) and is not greater than 10 when k is varied up to 10. The magnetic instability tends to occur at a higher wavenumber when k increases. For example, when k = 8, the critical azimuthal wavenumber is around m = 6. The growth rate tends to be higher at a higher wavenumber mode. The wave propagation is largely eastward and the phase velocity is not significantly different from the characteristic slow-wave speed. This study suggests that the geomagnetic westward drift seen at the low latitudes of the Earth's core surface is basically explained by a westward mean flow that is probably created by the thermal convection inside the core, but is partially modulated (slowed down) by the eastward wave propagation, if the instability of a strong toroidal field hidden below the core-mantle boundary is a significant process in the core dynamics.

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