A smoothing scheme for numerical solutions of the seismic wave equation

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In seismic wave propagation simulations for the long-period ground motion evaluation, a stable long-term integration is required for the structure model based on the shallow-deep integrated model. However, numerical instability often causes some divergences of calculation in practice. From experience it has been confirmed that some divergences of calculation often occur in the case that the spatial distribution of the structure has locally severe contrast. Therefore, as a method for mitigating numerical instability, it is reasonable to introduce a smoothing scheme in seismic wave propagation simulation. In order not to impair the characteristics of long-period ground motion by the smoothing scheme, it is desirable for the scheme to remove only spatially localized disturbance components of ground motion. In this study we discuss a smoothing scheme for numerical solutions of the seismic wave equation.

The smoothing scheme proposed in this research consists of both of the seismic wave equation and a correction term, which removes short wavelength components of ground motion selectively. The correction term for the smoothing scheme was derived heuristically by formally extending the operation of upwind difference method of advection equation, which is a stabilized method of the advection equation, to the one dimensional wave equation. As a result, we found that the derived correction term is an operator represented by a combination of the Laplacian and the heat equation. In what follows, we refer to the proposed smoothing scheme as a modified equation scheme. The modified equation scheme has the following features:
(a) It preserves the characteristics of the wave equation (wave propagation speed).
(b) It removes short wavelength components of ground motion selectively.
(c) It decreases energy moderately after short wavelength components of ground motion are removed.
In this study, we reveal that the modified equation scheme have the features above by numerical experiments and mathematical consideration of discretization method for the one dimensional wave equation. We also show that it is necessary to properly set a parameter related to the correction term.

Since the correction term added in the modified equation scheme is simple, we can easily apply it to two dimensional or three dimensional wave equation and more general seismic wave equations. By applying the modified equation scheme to two dimensional wave equation and seismic wave equation, it turned out that the modified equation scheme is also available for more realistic problems.

Keywords: seismic wave equation, smoothing scheme, long-period ground motion evaluation

