

## Earthquake magnitude and moment magnitude: Apparent fracture energy and damage zone thickness of faults

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**Introduction:** The relationships between the seismic energy  $E_s$  and the magnitude  $M_s$  and between the seismic moment  $M_o$  and the moment magnitude  $M_w$ , respectively, are usually expressed by the following equations.

$$\text{Log } E_s = 1.5M_s + 4.8 \quad (1)$$

$$\text{Log } M_o = 1.5M_w + 9.1 \quad (2)$$

Hereafter, we call the earthquake that almost follows these equations as the standard earthquake.

$M_s$  is smaller than  $M_w$  for the relatively large earthquakes along the Japan Trench. In the previous conference, Yamamoto shows the possibility that this difference is elucidated in terms of the seismic efficiency  $\eta$ . On the other hand,  $(M_j - M_w) > 0.2$  is found for many intra-earthquakes. For example,  $(M_j, M_w)$  is (7.3, 6.9) for the 1995 Hyogo ken Nanbu Earthquake, (7.3, 6.8) for the 2000 Tottori ken Seibu Earthquake, and (7.2, 6.8 to 6.9) for the 2008 Iwate-Miyagi Nairiku Earthquake. For these earthquakes,  $(M_s - M_w) > 0.2$ , provided  $M_j = M_s$ . According to the uniform DMF-model,  $M_w > M_s$  when the seismic efficiency  $\eta$  is 0.8 or less,  $M_w$ s for  $\eta > 0.8$ , and  $(M_s - M_w) = 0.2$  at  $\eta = 1$ , for the standard earthquake. This implies that the earthquakes of  $M_s - M_w > 0.2$  cannot be explained as the standard earthquake. In the present study, we will investigate the causes of  $(M_s - M_w) > 0.2$  besides  $\eta$ .

**Apparent fracture energy:** The damage zone fault (DMF)-model means that a fault has a finite thickness and consists of a damage zone (DZ) area and an asperity (AS) area. In the model, as a slip plane propagates in a DZ area, rotation of the DZ takes place. The apparent fracture energy is equivalent to the work done against the normal stress by the normal displacement to the fault plane due to the rotation. It is assumed that AS has the same rigidity  $\mu$  and shear fracture strength  $t_f$  as the matrix and that DZ is completely relaxed. The uniform DMF model means the fault that has infinite length and uniform thickness. The non-uniform DMF model does the fault with a finite length and non-uniform thickness.

**Results:** From the uniform DMF-mode, the followings are obtained: 1)  $E_s = (\delta t / 2 \mu) M_o$  at  $\eta = 1$ . Here,  $\delta t$  is the stress drop amount. 2) The fraction  $\phi$  of AS area in the fault plane is determined only by the critical strain  $e_f = t_f / \mu$  and  $\eta$ . Thus,  $\phi = (2x e_f)$  and  $\delta t$  is  $(2x e_f) \times (t_f)$  at  $\eta = 1$ . When  $e_f$  is set to  $1.5 \times 10^{-2}$ ,  $(M_s - M_w)$  becomes approximately 0.2 at  $\eta = 0.8$ . A larger value of  $t_f$  may be a possible explanation of  $(M_s - M_w) > 0.2$  in the case of uniform DMF-model.

Generally, a fault zone has a finite length and non-uniform thickness. The slip propagation requires the stress concentration at the tip of slip plane. It is assumed that the slip amount required for the slip propagation is equal to that of the critical weakening displacement that is necessary for the fracture of AS, and that the slip amount is constant throughout DZ. It is found that as the DZ thickness decreases to zero, the apparent fracture energy approaches twice the strain energy accumulated in AS, per unit area. This suggests that the slip propagation most likely is suppressed when the tip of the slip goes into the DMZ thinner than that around AS. This makes the fault area small compared with that of a standard earthquake for the same AS size, and this makes  $\delta t$  larger. This is another explanation.

**Conclusion:**  $(M_s - M_w) > 0.2$  occurs in the following cases. 1) For a standard earthquake, the seismic wave efficiency is 0.8 or more. 2) The average stress drop amount is larger than that of a standard earthquake. The larger stress drop amount is caused by the reasons as follows 2-1) the shear fracture strength of AS is larger than the case of standard earthquake, 2-2) the damage zone thickness is small outside the AS area. This means a case where the fault plane is geometrically non-uniform. The reason for the  $(M_s - M_w) > 0.2$  in the inland earthquake is that the earthquakes occur on the faults of more complicated structure than the

faults at the plate boundary. In order to draw a conclusion, it is necessary to know the structure of a fault, especially around its tips.

Keywords: fracture energy, seismic efficiency, magnitude, moment magnitude, damage zone fault model , critical weakening displacement