A Bayesian hierarchical model for a seismic source inversion

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The earthquake rupture processes of earthquakes are diverse reflecting the heterogeneous distribution of fault constitutive properties and stress, and fault geometry. Kinematic source parameters constrained by data provide the information related to the fault rupture dynamics and statistical properties of rupture processes. It is also used as an input for the calculation of Coulomb stress changes, the calculation of tsunami waveforms and the estimation of slip deficit.

A source inversion analysis using spatio-temporal displacement field data is able to be formulated as a discrete linear inverse problem when the Green function and the source fault are known. However, it is difficult to calculate an accurate Green function due to the lack of the accurate Earth structure model. Effects of the uncertainty of the Green function on the result of an inverse analysis increases with the improvement of the quality and the increase of the amount of data. Some of previous studies approximated the effects of the uncertainty of the Green function by introducing a new correlated and/or uncorrelated error term, which is added to data. The approximation, however, fails to capture important characteristics of the effects of the uncertainty of the Green function such as peak shift and heavy tails of the likelihood function under the uncertainty of the Green function.

To address the issue, we propose a hierarchical Bayes model for a seismic source inversion analysis. The model is targeted to the multi-data analysis with the multi-time-window finite fault source parameterization. We assume Gaussian observation errors and a Gaussian prior distribution for model parameters. In the model, a Green function is treated as a realization of a random variable G. We approximate the posterior distribution of the model parameters by approximately marginalizing G and the hyperparameters, which control the variance of the prior distribution of the model parameters and the variance of the observation errors. The marginalization of the hyperparameters is approximated by plugging in the maximum a posteriori hyperparameters given G. The marginal likelihood function for G is approximated by the Laplace approximation to the conditional posterior distribution of the hyperparameters given G. The marginalization of G is approximated by a Monte-Carlo integration method. The precisions of the two approximations are able to be improved by increasing the amount of data and number of samples of G.

We applied the method to synthetic data for far-field vertical P-wave displacement waveforms. We set a 1-D velocity structure consisting of two layers at the source region. We set the P-wave velocity, the S-wave velocity and the density of each layer to follow uniform distributions. We also set the depth of the interface of the layers to follow a uniform distribution. We made the true velocity structure by randomly sampling the 25th percentile point or the 75th percentile point for each random variable. We drew thousands of velocity structure samples from the probability density function (PDF), and then calculated the corresponding Green functions. For reference, we also conducted a conventional inversion, which used only a reference velocity structure (50th percentile point of the PDF of the velocity structure). We found that the conventional inversion result suffers artifacts especially at the later shallow part of the rupture process, while the result with the proposed method does not suffer from the artifact. Note that the mitigation of the artifact was not possible with the simple mean-of-the-posterior-mean approach. We also found increase of the Variance of the posterior distribution of the potency due to the marginalization of *G*.

The method could be a reasonable counterpart for the recent increase of Bayesian approaches with Monte-Carlo methods to study velocity structures.

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