

# Discrete Fourier transform spectroscopy using precisely periodic THz pulse train

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Fourier transform spectroscopy (FTS) is a spectroscopic technique where spectra are obtained by measuring a temporal waveform or interferogram of electromagnetic radiation, or other types of radiation, and calculating its Fourier transform (FT). FTS possesses inherent advantages over conventional dispersive spectrometers, such as a high signal-to-noise ratio, simultaneous acquisition of signals in a broad spectrum, and versatility for different radiation sources. Therefore, various types of FTS, including terahertz (THz) time-domain spectroscopy (THz-TDS), have been widely used in fields of research and industry.

When the temporal waveform of a phenomenon is measured, the spectral resolution is simply determined by the inverse of the measurement time window size during which the temporal waveform is observed. Therefore, as the time window size is extended, the spectral resolution is enhanced. However, when the majority of the signal components are temporally localized, excessive extension of the window size increases the noise contribution as well as the acquisition time. Furthermore, the travel range of a translation stage used for time-delay scanning practically limits the spectral resolution. On the other hand, when the phenomenon repeats, it is generally accepted that the achievable spectral resolution is limited to its repetition frequency because the maximum window size is restricted to a single repetition period to avoid the coexistence of multiple signals. If the spectral resolution of FTS can be improved beyond the repetition frequency, the scope of application of FTS will be further extended. In this paper, we demonstrate a significant spectral resolution improvement over the time window size limitation by using discrete Fourier transform spectroscopy (dFTS) with the mode-locked THz pulse train [1].

First we consider the measured temporal waveform  $h(t)$  of a phenomenon and its FT spectrum  $H(f)$  given by

$$H(f) = \int_{-\infty}^{\infty} h(t) \exp(-2\pi i f t) dt \quad (1).$$

This equation indicates that a spectral component  $H(f)$  is obtained by multiplying  $h(t)$  by a frequency signal  $\exp(-2\pi i f t)$  and then integrating the product for an infinite integration period. This process is illustrated in Fig. 1(a), where  $\cos 2\pi f t$  is shown as the real part of  $\exp(-2\pi i f t)$ . Although the achieved spectral resolution is infinitesimal in Eq. (1), the practical resolution is limited by the achievable finite integration period due to the SNR, the acquisition time, and/or the stage travel range.

Next we consider the case where  $h(t)$  is made to repeat by using precisely periodic pulsed radiation with a repetition period of  $T$ . When the relaxation time of  $h(t)$  is longer than  $T$ , a series of signals  $h(t)$  temporally overlap, each subsequent event being delayed by an integer multiple of  $T$  [see Fig. 1(b)]. When the time series data is acquired with a finite time window size  $\tau$ , the observed time window includes signal contributions from multiple periods, for example signals (A), (B), (C), and (D), as depicted in Fig. 1(c). To obtain the FT spectrum of these, the signals (A), (B), (C), and (D) are summed and multiplied by  $\exp(-2\pi i f t)$  before being integrated over  $\tau$ , as shown in Fig. 1(d). Here, if  $T$  is sufficiently stable and is exactly matched to  $\tau$  by ASOPS-THz-TDS [2], signals (A), (B), (C), and (D) can be temporally connected to form a single, temporally continuous signal alongside  $\exp(-2\pi i f t)$  [see Fig. 1(e)]. Despite the finite time window size ( $= \tau = T$ ), this connection is equivalent to acquiring the temporal waveform of  $h(t)$  without the limitation of the time window size, resulting in an infinitesimal spectral resolution at discrete frequencies. Furthermore, the combination of dFTS with the spectral interleaving [3] enables the large reduction of sampling interval. In the presentation, I will demonstrate the validity of the proposed method in low-pressure gas spectroscopy.

[1] Optica **2**, 460-467 (2015). [2] Appl. Phys. Lett. **87**, 061101 (2005). [3] Sci. Rep. **4**, 3816 (2014).

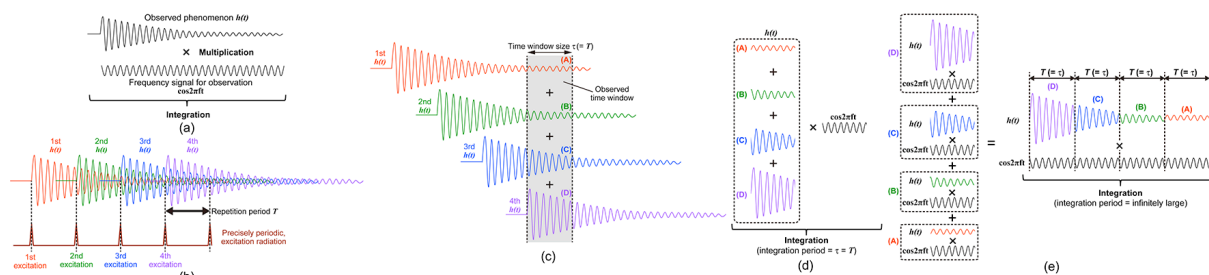


Fig. 1. Principle of operation.