Exceptional Line Formation and Flip-of-States in Degenerate Optical Microcavities

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1. Introduction

In the context of non-Hermitian systems *flip-of-state* phenomenon associated with an encircled second order exceptional point (*EP*) have attracted considerable attention [1, 2]. With an adiabatic variation of the chosen control parameters around an *EP* in parameter space, the corresponding coupled eigenvalues can be permuted in eigenvalue plane, exhibiting *EP* as a second order branch point. However, for an *EP* outside the closed loop, corresponding eigenvalues retain their initial positions. Such phenomenon exploiting continuous tuning of parameters via multiple *EP*s is yet to be explored. In this paper we propose a special line, coined as *exceptional line* that connects the successive *EP*s in parameter plane by virtue of the coupling parameters. Exploiting the interplay between the *EP*s on cascaded state-flipping mechanism along exceptional line we report the robustness of such phenomenon in presence of multiple *EP*s inside/outside the contour described by smooth /fluctuating variation of coupling parameters.

2. Design of microcavity and numerical results

Towards our goal, we design a 10μm 1D non-Hermitian Fabry-Parot type optical cavity with spatially imbalanced gain-loss as shown in Fig. 1(a) [3]. Using *S*-matrix formalism, we have identified five arbitrary pairs of poles, as an analogy to the complex eigenvalues of the associated Hamiltonian, for a chosen frequency range. With introduction of non-uniform gain-loss, a chosen pair of poles is mutually coupled. Adjusting the factor τ over the gain co-efficient (γ) up to 0.1, five different *EP*s are embedded and plotted in the (γ, τ)-plane as shown in Fig. 1(b). The best fitted curve indicates the formation a special single line, supporting all those *EP*s, proposed as exceptional line. Now, exploiting the benefit of exceptional line, via switching between them with proper tuning on the coupling parameters we have exemplified the flip-of-state phenomenon following the encirclement [1] of single or multiple *EP*s. In complex *k*-plane, trajectories represents evolution of two eigenvalues associated with an *EP*, followed by the anticlockwise evolution of on the described contours in (γ, τ)-plane.

Accordingly, for study of mutual dependence on eigenvalue dynamics between two nearby *EP*s, we display the eigenvalue behavior corresponding to EP_2 for two different system configurations with respect to EP_1. EP_2 is located outside or inside the chosen contours around the EP_1 as shown in Fig. 1(c)-[i] & [ii] for two different radii respectively. While considering the first case for two successive cycles around EP_1 in (γ, τ)-plane both eigenvalues corresponding to EP_2 return back to their initial position forming individual loops in *k*-plane. Whereas for the second case they form a complete loop after second permutation.

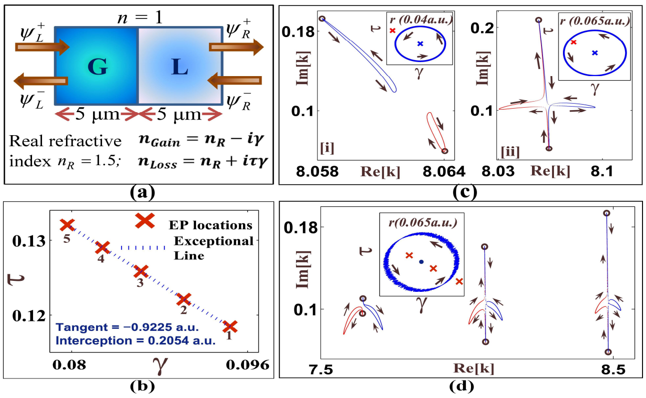

Fig. 1: (a) Proposed optical micro-cavity; **(b)** Fitted exceptional line supported by 5 nearest *EPs* (red crosses) with labels ; Trajectories of the eigenvalues (red and blue dotted line) starting from their respective initial positions (brown circles) corresponding to: **(c)** EP_2 (red cross) for its two different positions i.e. [i] outside and [ii] inside with respect to the contour around EP_1 (blue cross); **(d)** first three consecutive *EPs* for a common contour with deliberate fluctuations (blue circular path) around an arbitrary centre (blue dot) following the exceptional line. Radii are given at insets.

To analyze the stability of such state-flipping mechanism against any unwanted deformation in the encircling parameters, we study the eigenvalue behavior of three consecutive *EP*s in Fig. 1(d). The results are shown for a particular set of encircling parameters with deliberate fluctuations relative to an arbitrary center following the exceptional line such that EP_2 and EP_3 are present inside the loop and EP_1 stays outside the loop. After two successive encirclements in parameter plane, the eigenvalues corresponding to EP_2 and EP_3 form a closed loop in *k*-plane after second permutation in a very generic fashion, whereas the eigenvalues corresponding to EP_1 form individual loops avoiding any permutation.

3. Conclusions

Designing the unconventional microcavity, we have reported for the first time the formation of exceptional line supported by multiple *EP*s. Via suitable tuning of coupling parameters along the exceptional line we have exemplified the robustness of state-flipping mechanism associated with *EP*s. We have established that as long as we encounter an *EP* inside the enclosed loop, the flip-of-state mechanism is omnipresent irrespective of any moderate fluctuations. Novel cavity performance based on such robust scheme may open up a new class of integrated photonic devices.

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References

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