A study on the Modulational Instability in delayed Saturable Nonlinear Optical Media

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1. Introduction

One of the most advanced frontiers of the nonlinear optics is the Modulation instability (MI) [1]. MI is a good old phenomena featuring in most nonlinear wave systems in nature. It is the interplay between the self-phase modulation (SPM) and group velocity dispersion (GVD), where weak perturbations typically the noise imposed on a CW state grow exponentially, as a result of the interplay between nonlinear and dispersive effects. Modulational instability has already been extensively studied for the case of instantaneous kerr medium, but as the input power reaches a threshold then higher order nonlinearity will inevitable comes which eventually saturates the nonlinear response (SNL) of the medium. Recently, Liu et al, deals the role of delayed nonlinear response with the effect of GVD, but till date there is no convincing report regarding the effect of HOD [2]. Thus, the combination of saturation and delay is an interesting problem and deserves our attention. Hence in this report we investigate the interplay between delay and saturation in the MI.

2. Model Equation

The dynamical equation governing the evolution of field envelope for the non-instantaneous nonlinear medium can be consider in a relaxational model as follows [3]

$$i \frac{\partial E}{\partial z} = \frac{\beta_2}{2} \frac{\partial^2 E}{\partial t^2} + i \frac{\beta_3}{6} \frac{\partial^3 E}{\partial t^3} - \frac{\beta_4}{24} \frac{\partial^4 E}{\partial t^4} + \gamma \, NE$$

$$\frac{\partial N}{\partial t} = \frac{1}{\tau}(-N + f(\Gamma, |E|^2))$$

Where, β_n is the nth order dispersion coefficient, N represents the nonlinear index of the medium and τ is the medium's response time.

2.1 Linear Stability Analysis

Performing the linear stability analysis of the above equation, one will arrive into the dispersion as follows

$$K = \sqrt{\left(\tilde{\gamma}^2 P_0^2 + \beta_2 \frac{\Omega^2}{2} + \beta_4 \frac{\Omega^4}{24}\right) - \tilde{\gamma}^2 P_0^2}, \qquad \tilde{\gamma} = \frac{1}{(1 + i\Omega)}$$

For the case of pulse propagating with the effect of GVD, the two typical cases of dispersion regime takes the form as $\beta_2 < 0$ and $\beta_2 > 0$, corresponding the anomalous (AD) and normal dispersion (ND) regime. The MI gain for the typical cases is portrayed and the influence of the delay in the response is portrayed in the **Fig.1**.
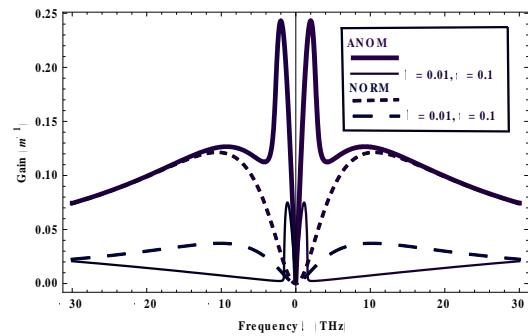
Fig..1 MI spectrum in the normal and anomalous dispersion regime for $(\beta_2 = \pm0.06, \beta_4 = \pm0.0007, P = 10W, \gamma = 0.015W^{-1}m)$

2.2 Discussion and Conclusion

We infer from the figures that MI occurs in both the regime irrespective of the sign of the dispersion. This is interesting because of the fact that the normal dispersion regime is not subjected to MI, due to the lack of phase matching between linear and nonlinear effects. However, the existence of MI band in ND regime is attributed to the fact that any finite delay in nonlinear response leads to imaginary part to the wave vector and thereby extends the MI even to the normal GVD. On the other hand, the saturation of nonlinearity suppresses the MI, due to the reduced value of the effective nonlinearity as it is evident from the Fig. 1. Thus delay extends the instability window whereas the saturation suppresses the MI. Thus the two opposite physical effects, saturation and relaxation makes the study interesting the context of MI.

References
[1] G. P. Agrawal, "Nonlinear Fiber Optics" third Edition, Academic Press, San Diego(2001).
[2] X. Liu *et al.,* "Modulational Instability for a relational Kerr medium,opt.comm.,281,2907(2008).