# A partial Mueller matrix polarimeter using two photoelastic modulator and polarizer pairs 

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In this paper，the theoretical approach is presented for measuring the partial Mueller matrix of a sample by using two photoelastic modulator－polarizer pairs，where the photoelastic modulators are operating at different resonant frequencies．The two PEMs in the instrument are oriented $45^{\circ}$ apart．From the given configuration，partial Mueller matrix in terms of the output light intensity of the light beam can be defined．

## 1．Introduction

Photoelastic modulators（PEMs）are commonly use in various kinds of polarization measurement techniques for many years．When a light beam passes through an optical element that is mechanically stressed by the acoustic wave，this is known as the basic working principle of photoelastic modulation［1］．In this paper，we described the design and theoretical approach to measure the Mueller matrix elements of a sample from the output light intensity of the light beam by using PEM－polarizer pairs．

## 2．Instrumentation

As shown in Figure 1，a system consists of a polarizer，PEMs operate at different resonant frequency（ 60 kHz and 50 kHz respectively） whereas the PEM $1(60 \mathrm{kHz})$ is oriented at $90^{\circ}$ and PEM $2(50 \mathrm{kHz})$ is oriented at $45^{\circ}$ ．
The light signal is recorded by photo detector digitized by an analog－to－digital converter．In the 2－PEM polarimeter，the polarization state generator（PSG）and polarization state analyzer （PSA）are composed of PEM－polarizer pairs respectively．
Light emitted by Laser－Driven Light Source （LDLS）which is used as a light source travels from left to right for transmission．


Fig． 1 Schematic diagram of PEM－polarizer pairs．

## 3．Theoretical Analysis

We used Mueller algebra to analyze the optical configuration shown in Figure 1．The Mueller matrix for the system is
$S_{\text {out }}=A\left(\theta_{2}\right) \cdot \operatorname{PEM}\left(\delta 2,45^{\circ}\right) \cdot M_{s} \cdot \operatorname{PEM}\left(\delta 1,90^{\circ}\right) \cdot P\left(\theta_{1}\right) \cdot S_{\text {in }}$
where A indicates a Mueller matrix of an analyzer，

PEM indicates the photoelastic modulator， $\mathrm{M}_{\mathrm{s}}$ is the Mueller matrix of sample and $S_{i n}$ is the input Stokes vector．
The instrument has a PEM－polarizer pair in both polarization state generator（PSG）and polarization state analyzer（PSA）．
By substituting Bessel function of the first kind and taking into account the limitations of the experimental setup，the detected output light intensity is given by

$$
\begin{align*}
I= & \frac{1}{4}\left[M_{00}+M_{01} \cos 2 \theta_{1}+M_{20} \sin 2 \theta_{2}+M_{21} \cos 2 \theta_{1} \sin 2 \theta_{2}\right. \\
& +\left(M_{02} \sin 2 \theta_{1}+M_{22} \sin 2 \theta_{1} \sin 2 \theta_{2}\right)\left(2 J_{2}(\mathrm{~A}) \cos \left[2\left(\omega_{1} t\right)\right]\right) \\
& +\left(M_{03} \sin 2 \theta_{1}+M_{23} \sin 2 \theta_{1} \sin 2 \theta_{2}\right)\left(2 J_{1}(\mathrm{~A}) \sin \left(\omega_{1} t\right)\right) \\
& +\left(M_{10} \cos 2 \theta_{2}+M_{11} \cos 2 \theta_{1} \cos 2 \theta_{2}\right)\left(2 J_{2}(\mathrm{~A}) \cos \left[2\left(\omega_{2} t\right)\right]\right)  \tag{2}\\
& +\left(-M_{30} \cos 2 \theta_{2}-M_{31} \cos 2 \theta_{1} \cos 2 \theta_{2}\right)\left(2 J_{1}(\mathrm{~A}) \sin \left(\omega_{2} t\right)\right) \\
& +M_{12} \sin 2 \theta_{1} \cos 2 \theta_{2}\left(2 J_{2}(\mathrm{~A})\right)^{2}\left(\frac{1}{2}\left\{\cos \left(2 \omega_{1}+2 \omega_{2}\right) t+\cos \left(2 \omega_{1}-2 \omega_{2}\right) t\right\}\right) \\
& +M_{13} \sin 2 \theta_{1} \cos 2 \theta_{2}\left(2 J_{1}(\mathrm{~A})\right)\left(2 J_{2}(\mathrm{~A})\right)\left(\frac{1}{2}\left\{\sin \left(\omega_{1}+2 \omega_{2}\right) t+\sin \left(\omega_{1}-2 \omega_{2}\right) t\right\}\right) \\
& -M_{32} \sin 2 \theta_{1} \cos 2 \theta_{2}\left(2 J_{1}(\mathrm{~A})\right)\left(2 J_{2}(\mathrm{~A})\right)\left(\frac{1}{2}\left\{\sin \left(2 \omega_{1}+\omega_{2}\right) t-\sin \left(2 \omega_{1}-\omega_{2}\right) t\right\}\right\} \\
& \left.-M_{33} \sin 2 \theta_{1} \cos 2 \theta_{2}\left(2 J_{1}(\mathrm{~A})\right)^{2}\left(\frac{1}{2}\left\{\cos \left(\omega_{1}-\omega_{2}\right) t-\cos \left(\omega_{1}+\omega_{2}\right) t\right\}\right)\right]
\end{align*}
$$

From the above Equation 2，the substitution of $\theta_{1}$ and $\theta_{2}$ at one orientation for both polarizer and analyzer will result in the measurement of nine Mueller matrix elements in a single measurement．

## 4．Conclusion

Some measures need to be taken into account in order to be able to measure all of the Mueller matrix elements．The other seven elements can be measured by changing the azimuthal angles of the PEM－polarizers［1］or by using four photoelastic modulators［2］．

## 5．References

［1］G．E．Jellison，et．al．：Applied Optics，36， 31 （1997）．
［2］Oriol Arteaga et．al．：Applied Optics，51， 20 （2012）．

