Optimization Strategy for Designing Computer-Generated Holograms and Diffractive Optical Elements

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1. Introduction

A computer-generated hologram (CGH) or diffractive optical element (DOE) is adapted in a free-space optical system to modulate the incident wavefront and then generate a diffractive pattern in a specified output plane [1]. The optical field function of the diffractive pattern is related to the modulated function of CGH/DOE through an operation based on the Fourier transform [2]. For simplicity, I assume the output function is the Fourier transform of the input function. Because of constraints (conditions) imposed on the input and the output functions, there is in general no solutions to exactly fulfill the constraints [3]. A good approximation to the solution is determined when the difference between the approximation and the solution is reduced as much as possible. To find an approximation with minimum difference or error, one needs to decide an optimization algorithm and a merit function of CGH performance which will be introduced in details in this presentation.

2. Optimization Algorithms

Optimization algorithms are used to find approximations of the solution in scientific and engineering problems that do not have analytic relationships between the inputs and the outputs [4]. Design of CGH and DOE is a typical problem needing optimization approaches to search an approximation which minimizes the difference due to the constraints applied to the input and the output. The phase retrieval problem, similar to the CGH/DOE design, also needs optimization approaches to fulfill the physical conditions applied on the object and the observation image. Although many optimization algorithms have been developed for the last 50 years, new optimization algorithms are in a great demand for the CGH/DOEs of millions of pixels due to high performance manufacture techniques and electronic devices recently developed.

Many optimization methods have been developed and applied to design of DOE, including Gerchberg-Saxton algorithm (GSA), error diffusion method, direct binary search (DBS) method, simulated annealing (SA) algorithm, generic algorithm (GA), and hybrid methods. They are categorized into the direct and inverse methods. A direct method begins with perturbations in the CGH domain, in which CGH pixels or element parameters are selected to change. DBS, SA, GA, and error diffusion approaches belong to the direct optimization method. A typical inverse method is GSA, or called error-reduction (ER) or iterative Fourier-transform algorithm (IFTA), which is more effective than the direct methods. The main drawback is the poor image quality of the resultant CGH. In the talk, I will present two more inverse methods with pixelwise operations which significantly improve the quality of the diffractive image.

2. Merit Function

Merit function is critical in the optimization processing even before the optimization evolution begins. A merit function is a combination of weighted device parameters (indices) to identify the approximate solution of minimum error. Frequently used parameters include the root-mean-squared error, the diffraction efficiency, the signal-to-noise ratio (SNR), and the signal variation of the diffractive image. They help to determine the minimum error and thus the best approximation in the optimization process.

Merit function also decides the evolutionary direction in most direct algorithms. I categorize the parameters into the early blooming (high sensitive) and the late blooming (low sensitive) types, depending on their definitions. For example, the SNR can be normal SNR (early blooming type) or the peak SNR (late blooming type). One should use them with care. In addition, I will present the solution spaces of 3-pixels CGHs to illustrate the significance of the performance parameters and the merit function in determination of the final results.

3. Summary

To obtain a solution to CGH/DOE with minimum error in a free-space optical system, one needs to select an optimization algorithm and realize the details of the algorithm based on the merit function of performance parameters. The direct and the inverse optimization algorithms for designing CGH/DOE were introduced in the presentation, and the properties of the merit function of performance were illustrated. And, then a strategy for the optimization evolution of CGH/ DOEs was made.

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References

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