Tying Knots in a Quantum Fluid

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1. Introduction

Knots are familiar entities that appear at a captivating nexus of art, technology, mathematics, and science. They have recently attracted significant experimental interest, in contexts ranging from knotted DNA [1] and nanostructures [2] to nontrivial vortex knots in classical fluids [3]. Within classical field theories [4], knots have been proposed as the basis of fundamental particles, as well as explaining diverse persistent phenomena such as atoms and molecules [5].

We describe here the first controlled experimental creation and detection of knot solitons [6], which are particle-like topological excitations possessing a knotted field character [7-9]. The superfluid medium is a Bose-Einstein condensate at a temperature some tens of billionths of a degree above absolute zero. In addition to enabling future experimental studies of their properties and dynamics, these knot solitons provide a striking demonstration of the celebrated Hopf fibration [10], which mathematically ties together many seemingly unrelated physical phenomena.

2. The Knot Soliton

A knot soliton is a topological structure that exists in a uniform background field consisting of vectors that all point in the same direction. The soliton itself consists of a continuous, three-dimensional orientation of these vectors in such a fashion that all of the vectors cannot be restored to the uniform configuration by simple rotations alone. Every vector orientation is represented within the soliton, with common orientations lying on closed curves in real space, and each of these curves is linked with every other exactly once. One may express this as a nontrivial mapping from real space to the space of unit vectors, $R^3 \sim S^3 \rightarrow S^2$, discovered mathematically by Hopf in 1931 [10]. Our experiment realizes this mapping by creating the appropriate vector field $\mathbf{d}(\mathbf{r})$ [9].

3. The Quantum Fluid

We work with a Bose-Einstein condensate of Rubidium-87 atoms. This superfluid is a highly controlled, coherent environment within which to create topological excitations, including quantized vortices [11] and monopoles [12]. The spin-1 condensate is described in terms of a macroscopic wavefunction Ψ, which consists of a scalar part expressing atomic density and phase that multiplies a three-component normalized spinor. Within the "polar" magnetic phase, the spinor may also be expressed in terms of a unit vector $\mathbf{d}(\mathbf{r})$ within which the knot structure is inscribed. Initially, $\mathbf{d}(\mathbf{r})$ points along the z axis everywhere.

3. Tying the Knot

The experiment begins when we suddenly apply a non-uniform magnetic field, causing the vectors $\mathbf{d}(\mathbf{r})$ to undergo Larmor precession. The appropriate orientation of $\mathbf{d}(\mathbf{r})$ emerges after approximately 500μs.

4. Detecting the Knot

The knot is detected after releasing the superfluid from its confinement and photographing the atomic density in each of its three spin components. The resulting patterns clearly reveal the expected linked rings, and are in excellent agreement with numerical simulations.

5. Conclusions

We have created and detected a knot soliton, the first in a quantum-mechanical medium.

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