Role of sparsity in optical phase imaging

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1. Introduction

Phase measurement is an important problem in Optics with wide ranging applications including large scale interferometers (e.g. LIGO), imaging of transparent cells for diagnostic applications, optical metrology, astronomical imaging through atmosphere, coherent diffraction imaging with X-rays, to name a few topics of interest. Phase measurement systems are either interferometric or non-interferometric. In either case phase reconstruction is inherently a computational problem where the laboratory measurements involve detection of light intensity which is real positive valued but the mathematical solution we seek is complex valued. Over the last few years, phase reconstruction problems are increasingly being treated as optimization problems. Our work involving sparsity based optimization has for example demonstrated several results where we can readily and practically surpass traditional resolution and noise limits on phase measurement leading to improved devices [1-6]. In this paper I will briefly discuss the optimization framework that allows us to surpass several traditional textbook limits on resolution and noise in phase imaging.

2. Formulation of optimization problem for phase reconstruction in terms of Wirtinger derivatives

The optimization formulation of phase reconstruction problems involves minimization of a cost function of the following form:

$$C(g,g^*) = \| D - S(g,g^*)\|^2 + \psi(g,g^*) \qquad (1)$$

Here D is the data vector consisting of intensity measurements and $S(g,g^*)$ is the data model for the complex valued solution g. The first term in Eq. (1) is L2-norm squared data fitting term and $\psi(g,g^*)$ denotes a penalty function that puts a physically desirable constraint on the solution. For example $S(g,g^*)$ denotes interference model in case of interferometric phase imaging or Fourier intensity model in case of non-interferometric phase retrieval. As will be explained in detail, we select penalty functions involving functions of gradients of g and g^* that encourage image sparsity. It will be shown that the minimization formulation as in Eq. (1) is equivalent to a standard Euler-Lagrange problem that has to be solved iteratively. The iterative solutions require calculation of the functional derivatives of the cost function. Unlike usual real variable optimization formalism discussed in most texts, the concept of complex or Wirtinger derivative [7] plays an important role in these problems.

3. Results and discussion

The sparsity based optimization model for image reconstruction is in principle different from traditional approaches like film based holography and allows us to overcome the limitations of traditional methodologies. For example I will show that it is possible to obtain full diffraction limited resolution from a single-shot off axis hologram even when the dc and cross terms overlap significantly in Fourier space. The proposed method also allows us to beat the traditional single pixel based shot-noise-limit on phase measurement with classical light. I will also demonstrate ROI reconstruction capability of this method for image plane digital holographic microscopy applications. Applications of this optimization based interferometric imaging methodology to diagnostic pathology and live cell imaging will be described.

For the case of non-interferometric phase retrieval for complex valued objects, sparsity ideas enable us to resolve the long standing twin-stagnation issue which arises since an object $g(x,y)$ and its twin $g^*(-x,-y)$ have the same Fourier magnitude.

4. Summary

Interferometric and non-interferometric phase imaging methodologies are required for a wide ranging applications and imaging devices. Incorporation of sparsity ideas in phase reconstruction problems allow us to question several traditional Physics limits on the performance of phase imaging systems. We therefore expect the proposed methodologies to have significant impact on design of phase measurement systems.

References

[1] K. Khare, P.T. Samsheer Ali, J. Joseph, Opt. Express **21** (2013) 2581.
[2] M. Singh, K. Khare, A. K. Jha, S. Prabhakar, and R. P. Singh, Phys. Rev. A **91** (2015) 021802 (R).
[3] M. Singh and K. Khare, JOSA A **34** (2017) 349.
[4] M. Singh, K. Khare J. Mod. Opt. **65** (2018) 1127.
[5] C. Gaur, B. Mohan, K. Khare, JOSA A **32** (2015) 1922.
[6] C. Gaur, P. Lochab, K. Khare, J. Opt. **19** (2017) 055703.
[7] D. H. Brandwood, IRE Proc. Microwaves and Antennas **130** (1983) 11.