# Localization of an atomic system in 2D subwavelength domain 

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## 1．Introduction

According to Werner Heisenberg，atom cannot be lo－ calized in a domain less than half of field＇s wavelength． Several approaches have been made to overcome this limi－ tation via incorporating the atomic coherences and quantum interferences．In order to this，researchers utilize the atomic energy levels and couple them with coherent standing－wave couple fields［1］．In case of standing－wave couple fields， spatial probe absorption provides the precise information of atom localization in subwavelength domain［2，3］．Atom localizes at positions where it has larger probe absorption． The motivation for present study is to find the atom locali－ zation for a realistic atomic system where nearby levels， thermal atomic velocity cannot be ignored．

## 2．Theoretical model and results

We consider $\Xi$－type atomic system with nearby upper levels as shown in Fig．1．This system is driven by a run－ ning probe $\left(G_{p}\right)$ and standing－wave couple（ $\left.G_{c}(\mathrm{x}, \mathrm{y})\right)$ fields．
（a）


Fig．1．（a）A detailed energy level diagram of $\Xi$－type atomic system by considering ${ }^{87} \mathrm{Rb}$ ．（b）Field configuration

The Hamiltonian of atom－light system after carrying out rotating wave approximation（RWA）is given by

$$
\begin{aligned}
H= & -\left(\Delta_{p}+\Delta_{c}-\delta_{2}\right)|5\rangle\langle 5|-\left(\Delta_{p}+\Delta_{c}+\delta_{1}\right)|4\rangle\langle 4|-\left(\Delta_{p}+\Delta_{c}\right)|3\rangle\langle 3|-\Delta_{p}|2\rangle\langle 2| \\
& -\frac{i}{2}\left[G_{p}|1\rangle\langle 2|+a_{32} G_{c}|2\rangle\langle 3|+a_{42} G_{c}|2\rangle\langle 4|+a_{52} G_{c}|2\rangle\langle 5|+H . c .\right],
\end{aligned}
$$

where $a$＇s and $\Delta$＇s are relative transition strengths and fields detuning between levels．The dynamics of at－ om－light system can be described using Louville equation given as

$$
\dot{\rho}=-\frac{i}{\hbar}[H \rho]+L \rho
$$

Imaginary part of $\rho_{21}$ provides the spatial probe absorption which is a key factor for determining the atom localization position．It directly reflects the conditional probability．

Case 1．Stationary atom：We first consider the case where $G_{c}$ create the standing wave along the $x$－and $y$－axis，i．e．， $G_{c}=\Omega(\operatorname{Sin} k x+\operatorname{Sin} k y)$ with $k=2 \pi / \lambda$ ，shown in Fig．1（b）．As
can be seen in Fig．2（a），contour plot shows that atom lo－ calizes only in two quadrants with high precision in posi－ tion．Therefore，the probability of finding atom at a position is only $1 / 2$ ．On addition of extra running－wave field $\left(g_{c}\right)$ in previous field configuration（i．e．，$G_{c}=\Omega(\operatorname{Sin} k x+\operatorname{Sin} k y)+$ $\mathrm{g}_{\mathrm{c}}$ ），it causes the atom to localize in only one quadrant［Fig． 2（b）］，i．e．，probability of finding atom become unity under specific parametric condition．This kind of atom localiza－ tion can be understood on the basis of dresses state picture．


Fig．2．Variation of $\operatorname{Im}\left(\rho_{21}\right)$ vs position（ $k x, k y$ ）．（a）$\Delta c=g_{c}=0$（b） $\Delta_{c}=2 \mathrm{~g}_{\mathrm{c}}=2 \Gamma_{21}$ ．The other parameters are $\Omega=5 \Gamma_{21}, \Delta_{\mathrm{p}}=9 \Gamma_{21}$ ， $\mathrm{G}_{\mathrm{p}}=0.001 \Gamma_{21}, \delta_{1} / 2 \pi=9 \mathrm{MHz}$ and $\delta_{2} / 2 \pi=7.6 \mathrm{MHz}$ ．

Case 2．Moving atom：The velocity of moving atom（v）can be integrated in numerical analysis by replacing $\Delta_{p}$ with $\left(\Delta_{\mathrm{p}}+k_{p} \mathrm{v}\right)$ due to Doppler effect．Therefore，thermal averag－ ing of $\rho_{21}$ has been carried out with Maxwell－Boltzmann velocity distribution．Fig． 3 shows the atom localization in sub－wavelength domain for moving atom in absence（Fig． 3a）and presence（Fig．3b）of nearby upper levels．One can conclude here that it is hard to find exact localization posi－ tion of a moving atom as absorption maxima distributed all over the subwavelength domain．


Fig．3．Variation of $\operatorname{Im}\left(\rho_{21}\right)$ vs position（kx，ky）．（a）Without nearby levels，（b）With nearby levels．The other parametric conditions are same as in Fig．2（a）．

## 3．Conclusions

Atom localization has been discussed for $\Xi$－type atom－ ic system under the influence of nearby levels as well as of thermal motions．Thermal motion of atomic system results in lowering in precision of atom position．

## References：

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