# Multiple Scale Analysis of Vertical Silicon Slot waveguide for Nonlinear Applications 

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## 1．Introduction

Recently proposed，slot optical waveguide is the great interest of topic to explore the capability of optical signal processing because CMOS fabrication technology allows the fabrication of the device in order of nanometer．Slot waveguide has extensively explored for the applications of sensing，switching，modulator，and optical interconnects［1］． The electric field confined in the low index slot region which is created by sandwiched the low index material by two sili－ con slab waveguide．The low index material such as a nonlinear polymer or an organic material can be integrated by silicon waveguide and become a silicon organic hybrid （ SOH ）slot waveguide［2］．Then，confined high electric field in low index region employed to the nonlinear optical signal processing in SOH slot waveguide．

In this paper，analysis of slot waveguide has been pre－ sented with an analytical method called＂Multiple scale method＂［3］．It is a combination of the effective index method with perturbation correction technique for planar waveguide．The perturbation in the effective index has ac－ quired for confined optical power which is responsible for nonlinear phase change of propagating electric field in slot waveguide．

## 2．Theory

For analysis of a nonlinear slot waveguide，the refractive index due to nonlinear effect in the slot is modified as $n_{n l}=$ $n_{s o}\left[1+\frac{n_{2}|E|^{2}}{z_{0}}\right]^{\frac{1}{2}} \quad[2]$ ．Using the perturbation technique within the slot waveguide Eigen value equations for $0^{\text {th }}$ and $1^{\text {st }}$ order are obtained by the solving the $0^{\text {th }}$ and $1^{\text {st }}$ order boundary condition for electric field and magnetic field in slab－slot waveguide boundaries．The propagation constant $\left(\beta_{0}\right)$ of slot waveguide without any perturbation is calculated through $0^{\text {th }}$ order Eigen value equation．The perturbed refrac－ tive index of slot waveguide is calculated through the $1^{\text {st }}$ or－ der perturbation terms as $\beta_{1}$ ．

$$
\beta_{1}=\frac{\left(\frac{n_{f}^{2}}{\kappa_{f}}\right) P_{2} Q_{2}+P_{1} Q_{1}}{P_{1} T_{1}+\left(\frac{n_{f}^{2}}{\kappa_{f}}\right) P_{2} T_{2}+\left(\frac{\gamma_{c}}{n_{c}^{2}}\right) T_{3}+T_{4}}
$$

The terms of the above equation is given in Appendix． For perturbed slot waveguide，the propagation constant is calculated as $\beta=\beta_{0}+\delta \beta_{1}$ ．The nonlinear effective index is $\frac{\beta}{k_{0}}\left(=\frac{\beta_{0}-\delta \beta_{1}}{k_{0}}\right)$ and linear effective index is $\left(\beta_{0} / k_{0}\right)$ ．

## 3．Conclusions

In summary，multiple scale method is presented for analysis of the nonlinear silicon slot waveguide．The modi－ fied effective index is obtained due to nonlinear perturbation in refractive index of slot．This analysis can be used for the－ oretical modelling of the nonlinear optical process．

## References

［1］V．R．Almeida，et al．，Opt．Lett． 29 （2004） 1209.
［2］T．Vallaitis，et al．，Opt．Express． 17 （2009） 17357.
［3］V．Priye，et al．，J．Comput．Electron． 17 （2018）857－865．

$$
\begin{aligned}
& \text { Appendix } \\
& T_{1}=-2 \beta_{0} a \sinh \gamma_{s} a\left[\frac{1}{2 \gamma_{s}}+\frac{1}{2 \kappa_{f}}\left(\frac{n_{f}^{2}}{n_{s 0}^{2}} \frac{\gamma_{s}}{\kappa_{f}}\right)\right] \\
& T_{2}=\left(-\frac{2 \beta_{0}}{2 \gamma_{s} n_{s o}^{2}}\right)\left(\sinh \gamma_{s} a+\gamma_{s} a \cosh \gamma_{s} a\right)-\left(\frac{2 \beta_{0}}{2 \kappa_{f} n_{f}^{2}}\right)\left(\frac{n_{f}^{2}}{n_{s o}^{2}} \frac{\gamma_{s}}{\kappa_{f}}\right) \sinh \gamma_{s} a \\
& +\left(\frac{\beta_{0} a}{n_{f}^{2}}\right) \cosh \gamma_{s} a \\
& T_{3}=\left(-\frac{2 \beta_{0} b}{2 \kappa_{f}}\right) \sin \kappa_{f}(b-a) \cosh \gamma_{s} a \\
& +\left(\frac{2 \beta_{0} b}{2 \kappa_{f}}\right) \cos \kappa_{f}(b-a)\left(\frac{n_{f}^{2}}{n_{s o}^{2}} \frac{\gamma_{s}}{\kappa_{f}}\right) \sinh \gamma_{s} a \\
& -\left(\frac{2 \beta_{0} b}{2 \gamma_{c}}\right) K_{c}-\left(\frac{\kappa_{f}}{n_{f}^{2}}\right) B_{1} \sin \kappa_{f}(b-a) \\
& +\left(\frac{\kappa_{f}}{n_{f}^{2}}\right) B_{2} \cos _{f}(b-a)+\left(\frac{\gamma_{c}}{n_{c}^{2}}\right) \\
& T_{4}=\left(\frac{\beta_{0}}{\kappa_{f} n_{f}^{2}}\right)\left\{\cos \kappa_{f}(b-a)-\kappa_{f} b \sin \kappa_{f}(b-a)\right\} \times\left(\frac{n_{f}^{2}}{n_{s o}^{2}} \frac{\gamma_{s}}{\kappa_{f}}\right) \sinh \gamma_{s} a \\
& -\left(\frac{\beta_{0}}{\kappa_{f} n_{f}^{2}}\right)\left\{\sin \kappa_{f}(b-a)+\kappa_{f} b \cos \kappa_{f}(b-a)\right\} \\
& \times \cosh \gamma_{s} a+\left(\frac{\beta_{0}}{\gamma_{c} n_{c}^{2}}\right)\left(\gamma_{c} b-1\right) K_{c} \\
& Q_{1}=\left(\frac{a}{8 \gamma_{s}}\right) \frac{k_{0} \alpha\left|A_{0}\right|^{2}}{\left(\omega \epsilon_{0} n_{s o}^{2}\right)^{2}}\left\{3 \beta_{0}^{2}+\gamma_{s}^{2}\right\} \sinh \gamma_{s} a \\
& +\left(\frac{1}{32 \gamma_{s}^{2}}\right) \frac{k_{0} \alpha\left|A_{0}\right|^{2}}{\left(\omega \epsilon_{0} n_{s o}^{2}\right)^{2}}\left\{\beta_{0}^{2}+\gamma_{s}^{2}\right\} \cosh 3 \gamma_{s} a \\
& Q_{2}=\left(\frac{1}{8 \gamma_{s} n_{s o}^{2}}\right) \frac{k_{0} \alpha\left|A_{0}\right|^{2}}{\left(\omega \epsilon_{0} n_{s o}^{2}\right)^{2}}\left(3 \beta_{0}^{2}+\gamma_{s}^{2}\right) \\
& \times\left(\sinh \gamma_{s} a+\gamma_{s} a \cosh \gamma_{s} a\right) \\
& +\left(\frac{3}{32 \gamma_{s} n_{s o}^{2}}\right) \sinh 3 \gamma_{s} a \frac{k_{0} \alpha\left|A_{0}\right|^{2}}{\left(\omega \epsilon_{0} n_{s o}^{2}\right)^{2}}\left(\beta_{0}^{2}+\gamma_{s}^{2}\right) \\
& +\left(\frac{\gamma_{s}}{k_{0} n_{s o}^{4}}\right) \frac{k_{0} \alpha\left|A_{0}\right|^{2}}{\left(\omega \epsilon_{0} n_{s o}^{2}\right)^{2}}\left\{\beta_{0}^{2} \cosh ^{2} \gamma_{s} a \sinh \gamma_{s} a+\gamma_{s}^{2} \sinh ^{3} \gamma_{s} a\right\} \\
& P_{1}=-\frac{\kappa_{f}}{n_{f}^{2}} \sin \kappa_{f}(b-a)+\frac{\gamma_{c}}{n_{c}^{2}} \cos \kappa_{f}(b-a) \\
& P_{2}=-\frac{\kappa_{f}}{n_{f}^{2}} \cos \kappa_{f}(b-a)+\frac{\gamma_{c}}{n_{c}^{2}} \sin \kappa_{f}(b-a)
\end{aligned}
$$

