Hamiltonian ray tracing of compressed lens via transformation-optics

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1. Introduction

Today's technology is driving the need for lighter, simpler, and more compact optical devices [1]. As part of that, a design method using the transformation optics is actively researched. The transformation optics is a new paradigm for the science of light that is enabled by recent developments in metamaterials [2]. The material properties designed by the transformation-optic method can be supported by the fast advance in the field of metamaterials. By combining metamaterial with Hamiltonian optics, we propose an unprecedented type of lens for an imaging system. This transformation-optic designed lens is devised for a compact imaging system. In this paper, we study the optical characteristics of transformation-optic compressed lens which have properties difficult to find in nature.

2. Design of compressed lens

It is a remarkable fact that Maxwell's equations under any coordinate transformation can be written in an identical "Cartesian" form if simple transformations are applied to the materials, the fields, and the sources [3, 4].

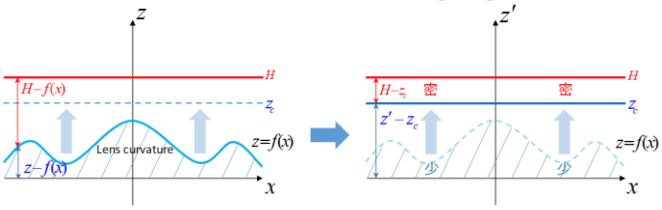

Fig.1. Schematic of lens compression

In Fig. 1, the shaded blue section is an aspherical lens surface. We have to spread this surface to Z_c to make it flat and we transform the coordinates to define a new coordinate system. Next, we need to calculate the permittivity and permeability function for each region. Z, a length of a perpendicular drawn from a point on an aspherical surface is given by equation (1).

$$Z = \frac{\frac{Y^2}{R}}{1+\sqrt{1-(1+K)\times\frac{Y^2}{R^2}}} + \sum_{i=1}^{aspherical\ order} A_{2i}\times Y^{2i} \qquad (1)$$

where Y and K are perpendicular height to the optical axis and conic constant. A_{2i} denotes an aspheric surface coefficient and R denotes radius of curvature of the aspherical surface. Let us consider the coordinate transformation from (x, y, z) to (x', y', z') described by equation (2) and (3).

$$\begin{aligned} (x',y',z') &= \left(x,\ y,\ \frac{z_c\cdot z}{f(x)}\right) && for\ 0\le z\le f(x) \\ (x',y',z') &= \left(x,\ y,\ \frac{(z-f(x))(H-z_c)}{H-f(x)}+z_c\right) && for\ f(x)\le z\le H \end{aligned} \qquad (2)$$

3. Hamiltonian and ray equations

The photon has an implicit relationship between its position and momentum. Therefore, if only the invariant function is defined in the ray-path section, it can be considered Hamiltonian. The Hamiltonian, which will be used for generating the ray paths has fundamentally the planar wave dispersion relation [5], as expressed in equation (3).

$$H = f(x)(\mathbf{knk} - \det(\mathbf{n})) \qquad (3)$$

where $f(\mathbf{x})$ is some arbitrary function of position. The equations of motion are

$$\begin{aligned} \frac{d\mathbf{r}}{d\tau} &= \frac{\partial H}{\partial \mathbf{k}} \\ \frac{d\mathbf{k}}{d\tau} &= -\frac{\partial H}{\partial \mathbf{x}} \end{aligned} \qquad (4)$$

where τ parameterizes the paths. The above, pair of coupled, the first-order differential equations for ray-tracing can be solved by the fourth-order Runge-Kutta method.

4. Conclusion

In this paper, we demonstrate that the transformation optic design method combined with metamaterial can make unusual optical lens possible. This lens is expected to contribute to a much compact imaging system.

References

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