# Random walk and toward lévy flight of light in waveguide lattice 

Sayan Bhattacherjee＊，Somnath Ghosh<br>${ }^{1}$ Department of Physics，Indian Institute of Technology Jodhpur，Rajasthan－342037，India<br>E－mail ：＊bsayan1992＠gmail．com

## 1．Introduction

Disorder causes the Bloch waves to localize in otherwise periodic lattices resulting in Transverse Anderson Localiza－ tion［1］．However，certain localization regime exists where the logarithm of the transverse width is proportional to log－ arithm of the propagation distance，known as Anomalous Localization．Such phenomenon occurs owing to enhance－ ment in non－uniformity in disordered system．In this regime due to random scattering，the photons mimic Lévy flight like random walk．Such distributions are identified by the heavy－tailed nature of the intensity fluctuation．This light dynamics can be observed in evanescently coupled wave－ guide arrays forming a lattice where scattering length can be tuned．These systems are fabricated out of a bulk material by etching out waveguides by using the technique of ULI（Ul－ trafast Laser Inscribtion）or lithography［2］．Here we report a very special kind of lattice where complex coupling has been patterned by selectively doping the bulk with active materi－ al．Controlled deliberate disorder introduced to waveguides helps to arrest transverse spreading of light and in presence of adequate optical gain，lattice exhibits anomalous locali－ zation which is reported for the first time．This has also been confirmed by calculating the random walk of the photons． Activity in the direction of random lasing through these structures can be explored by anomalous localization and Lévy－type distribution in intensity fluctuations［3，4］．

## 2．Design of the optical waveguide and results

The schematic of the proposed lattice is shown in Fig （a．1）．The arrow denotes the approximate location for the input Gaussian beam whose dynamics is being studied us－ ing scalar beam propagation equation under the paraxial approximation．Transverse refractive index variation of the lattices is mapped by the following equation：

$$
\Delta n(x)=\Delta n_{p}(x) H(x)+\Delta n_{p}(x) C \delta H(x)+(\mathfrak{J}(\Delta n(x)))
$$

The fluctuation of the material index along the transverse lattice direction is controlled by the first two terms of the equation，whereas the amount of active doping is manifest－ ed by the third part．The refractive index profile of central portion of the lattice Fig（a．2）shows a $10 \%$ disorder of the waveguide index． 300 waveguides of $7 \mu \mathrm{~m}$ thickness are embedded of higher refractive index at a separation of $7 \mu \mathrm{~m}$ on this bulk material．The non－hermitian nature of the lat－ tice arises from the active material being doped to the bulk． The simulation of light propagation through this lattice has been done assuming negligible band－gap effect at the oper－ ating condition．The transformation of the input beam into a quasi－localized state at the lattice end is evident from Fig （b．1）．It can be viewed from Fig（b．2）that the output inten－ sity fluctuation broadens to extend till the lattice edges in presence gain of 0.0001 ．


Fig．1：（a．1）Schematic of the proposed 1D waveguide lattice； where $\mathrm{n}_{1}$ and $\mathrm{n}_{2}$ represent r．i．of two different materials forming a bi－layer of width $14 \mu \mathrm{~m} ; \mathrm{W}$ is the total width of the lattice con－ sisting of 300 such bi－layers；$L$ is the total length of the lattice in the direction of light propagation，the arrow represents incident Gaussian light beam．（a．2）Transverse r．i．profile showing $10 \%$ disorder of the waveguide index value．（b．1）The ensemble aver－ age of spatial intensity distributions of the output light beam through a 10 mm long， $20 \%$ disordered passive lattice and（b．2）an active lattice having a gain of 0.0001 ．It shows the heavy－tailed nature of the transverse intensity profile．（c）Plot of possible ran－ dom walk of photons due to various disorders through 10 mm long passive lattice and $(\mathbf{d})$ active lattice（gain $=0.0001$ ）．Narrowing down of random walk phenomenon is the result of anomalous localization due to gain．

Now the lévy nature of random walk is calculated by taking the ratio of the expectation value of instantaneous state with respect to the total power，R．W．$=\left\langle\psi_{n}\right\rangle /\left|\psi_{n}\right|^{2}$ ． The trajectories in presence various disorder through pas－ sive and active lattice（gain $=0.0001$ ）are shown in Fig（c） and Fig（d）respectively

## 3．Conclusion

In summary，we have investigated the interplay of dis－ order and gain to an otherwise periodic lattice．The Lévy signature of the output intensity profile has been reported paving the path of further understanding of random lasers． It＇s also been concluded that in this regime the particles also follow Lévy flight like random walk due to scattering．

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