State-switching around a Fourth-order Exceptional Point in an Optical Microcavity

Arnab Laha,[1] Dinesh Beniwal,[2] Somnath Ghosh[1, *]

[1]Department of Physics, Indian Institute of Technology Jodhpur, Rajasthan-342037, India
[2]Department of Physics, National Institute of Science Education and Research Bhubaneswar, 752050, India
E-mail: *somiit@rediffmail.com

1. Introduction

Exceptional points (EPs) are the topological branch-point singularities in open (non-Hermitian) systems, where coupled eigenvalues and the corresponding eigenstates coalesce simultaneously [1-4]. At an n^{th}-order EP, n coupled states are analytically connected; that can also be realized with the simultaneous presence of $(n-1)$ 2^{nd}-order EPs (EP2s) [1-2]. An adiabatic parametric encirclement, enclosing an EP, results in the successive permutation among the corresponding coupled states, which was extensively studied in various open systems hosting EP2s [3-4] or EP3s (3^{rd}-order EPs) [1]. In this context, an analytical-treatment revealed the existence of 4^{th}-order EPs (EP4s) [2]; however, research on the realization of any real system hosting EP4s is lacking. In this paper, we report a gain-loss assisted tri-layer Fabry-Pérot type optical microcavity to host an EP4 with a simultaneous presence of three connecting EP2s, where successive state conversion among four coupled states around the embedded EP4 has been explored for the first time.

2. Design of the optical microcavity and results

We design a 1D tri-layer optical microcavity of length $L = 12\mu m$, where a layer of high-indexed ($n_s = 2.65$) material has been sandwiched between two thin layers of the same low-indexed materials ($n_g = 1.5$). Here, non-Hermiticity has been introduced with the onset of an unbalanced bi-layer gain-loss profile in both the low indexed regions, whereas in the intermediate region, there is no gain-loss. γ (gain-coefficient) and τ (loss-to-gain ratio) are two tunable control parameters to characterize the gain-loss profile. The overall complex refractive index profile can be written as

$$n(x) = \begin{cases} n_g - i\gamma, & \text{for } 0 \le x \le L_1 \text{ and } L_3 \le x \le L_4 \\ n_g + i\tau\gamma, & \text{for } L_1 \le x \le L_2 \text{ and } L_4 \le x \le L \\ n_s, & \text{for } L_2 \le x \le L_3 \end{cases} \quad (1)$$

The schematic of the designed microcavity (with all the operating parameters) has been shown in Fig. 1(a). The complex refractive index profile has been shown in Fig. 1(b) for a specific set of γ and τ. Using the scattering (S) matrix formalism method, we construct a 2×2 S-matrix associated with the designed microcavity and calculate the S-matrix poles using numerical root-finding method [3], where only the poles, appearing in the 4^{th}-quadrant of complex frequency (k) plane, gives the physical cavity resonances.

Now to study the fourth-order state-interaction phenomenon, we judiciously choose a set of four poles within the k-range from 2.8 μm to 3.6 μm. These poles are mutually coupled with the onset of the gain-loss profile. Investigating the avoided resonance crossings phenomena [3] between different pairs from the chosen set of poles, we identify three EP2s at (0.08, 0.62), (0.39, 0.88), and (0.33, 2.79) in the (γ,τ)-plane. Here, in between two EP2s, there must be a common pole. The second-order singular behavior of each of the EPs has been verified by encircling them individually in the (γ,τ)-plane for which the corresponding poles associated with a specific EP exhibit a flip-of-state phenomenon in complex k-plane. In this context, if we encircle any two EP2s, then we can observe a successive conversion among three corresponding poles. Now to study the functionalities of an EP4, we quasi-statically enclose three EP2s simultaneously in the (γ,τ)-plane following the parametric equations $\gamma = \gamma_0 \sin(\phi/2)$ and $\tau = \tau_0 - a\sin\phi$ with $0 \le \phi \le 2\pi$, $a = 2.2$, $\gamma_0 = 0.395$ and $\tau_0 = 0.95$. Following such an encirclement process, as shown in the inset of Fig. 1(c), the trajectories of all the chosen poles in the complex k-plane have been shown in Fig. 1(c). As can be seen here, four chosen poles exchange their identities from their initial positions, which reveals the presence of a 4^{th}-order branch point, i.e., an EP4 in the designed microcavity.

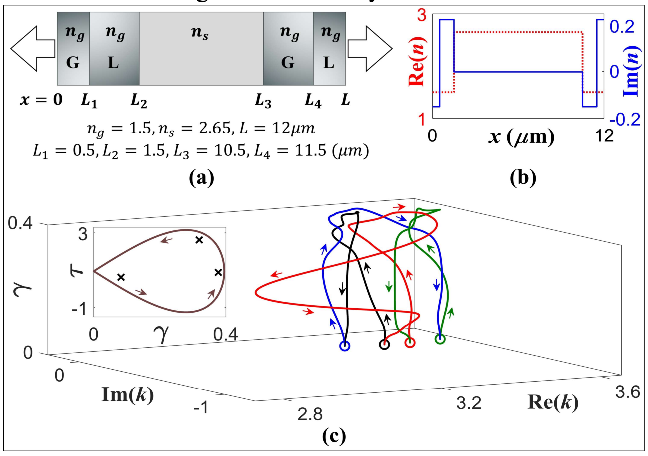

Fig. 1: (a) Schematic of the proposed microcavity (G → Gain, L → Loss). **(b)** Complex profile of $n(x)$ (given by Eq. 1), where Re(n) (red dotted line) and Im(n) (blue solid line; for $\gamma = 0.15$ and $\tau = 1.5$) correspond to left and right y-axes, respectively. (c) Successive conversion among four chosen poles in complex k-plane following a parametric encirclement process in (γ,τ)-plane, as shown in the inset (three black crosses represent three EP2s). Four circular markers of different colors indicate the initial locations (for $\gamma = 0$) of four poles, where the curves of respective colors indicate their trajectories. Arrows indicate the direction of progressions.

3. Conclusion

We report a successive conversion among four coupled states around an embedded EP4 with simultaneous presence of three connecting EP2s in a Fabry-Pérot type optical microcavity, which opens up a unique light manipulation tool.

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