Third-order Exceptional Point and State-switching in an All-lossy Microcavity

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1. Introduction: The interplay of gain-loss with the topology of an open optical system entitles the presence of exceptional points (EPs), a special type of topological defect in the parameter space, for which the corresponding coupled eigenvalues and the eigenvectors coalesce, simultaneously [1-3]. Lately, the effect of closed gain-loss variation around an EP2 has extensively been investigated to explore various applications associated with the topological state-transfer process [1-3]. However, the presence of gain beyond a particular threshold can accumulate excessive noise and additional gain-guided states, which can prevent the desired device performance. Such limitations can be overcome by designing an all-lossy system (gain-free). In this context, although an all-lossy waveguide system revealed the hallmark of 2nd-order EPs (EP2s) [4], the findings to explore the higher-order EPs in all-lossy systems are lacking. Here, we report the hosting of a 3rd-order EP (EP3) with the simultaneous presence of two connected EP2s in an all-lossy microcavity and explore the associated topological state-switching applications.

2. Design methodology and numerical results: A 1D trilayer open Fabry-Pérot type microcavity, occupying the region $0 \le x \le L$, has been designed by sandwiching a high-indexed (n_c) layer between two same low-indexed (n_g) layers. The associated scattering (S) matrix have been formulated, where the S-matrix poles appearing in the lower half of the complex frequency (k) plane physically reflects cavity resonance-states [1]. Instead of any gain, an unbalanced bilayer parameter-dependent loss profile has been introduced in two low indexed regions to couple the cavity states, where their interactions have been modulated by tuning two control parameters *viz.* a loss-coefficient (γ) and a fractional-loss-ratio (τ). The complex refractive index distribution (including the loss-profile) can be written as

$$n(x) = \begin{cases} n_g + i\gamma & : 0 \le x \le L_1 \text{ and } L_4 \le x \le L, \\ n_g + i\tau\gamma & : L_1 \le x \le L_2 \text{ and } L_3 \le x \le L_4, \\ n_c & : L_2 \le x \le L_3. \end{cases} \quad \textbf{(1)}$$

The schematic configuration of the microcavity along with the operating parameters and the profile of $n(x)$ (for a particular choice of γ and τ) have been shown in Figs. 1(a) and 1(b), respectively.

Three S-matrix poles have been selected within a chosen range of $\Re(k)$ within 6.6 to 7.8 (in μm^{-1}) to study their interactions (with the onset of loss-profile) toward the emergence of an EP3 with the simultaneous presence of two connected EP2s. Investigating the avoided resonance crossing type interactions among three poles with the variations of γ and τ, we have observed in Fig. 1(c) that Pole-1 and Pole-2 coalesce at $\gamma = 0.215$ for a specifically chosen $\tau = 0.121$, whereas Pole-1 and Pole-3 coalesce at $\gamma = 0.25$ for a specifically chosen $\tau = 0.188$. Hence, three chosen poles encounter two connected EP2s at (0.215, 0.121) and (0.25, 0.188) in the (γ, τ)-plane, which essentially refers to the presence of an embedded EP3 in the same parameter space associated with the unbalanced loss profile. To validate the 2nd-order branch-point behavior of each of the EP2s, we have considered individual quasistatic encirclements around them in the (γ, τ)-plane by following $\gamma = \gamma_c(1 + a_1 \cos\phi)$ and $\tau = \tau_c(1 + a_2 \sin\phi)$ with the proper choices of γ_c, τ_c, and $\{a_1, a_2\}$ over $0 \le \phi \le 2\pi$ and examined the associated adiabatic state-flipping processes in the complex k-plane. Eventually, to investigate the topological functionalities of an embedded EP3, we have tracked the trajectories of three poles in Fig. 1(d) by following a quasistatic loss variation around both the connected EP2s simultaneously (as shown in the inset; with $\gamma_c = 0.235$, $\tau_c = 0.15$, $a_1 = 0.2$, and $a_2 = 0.3$). Here, we have shown the successive switching process among three chosen poles, where they adiabatically permute their initial positions and exchange their identities, revealing the third-order branch point behavior of the embedded EP3.

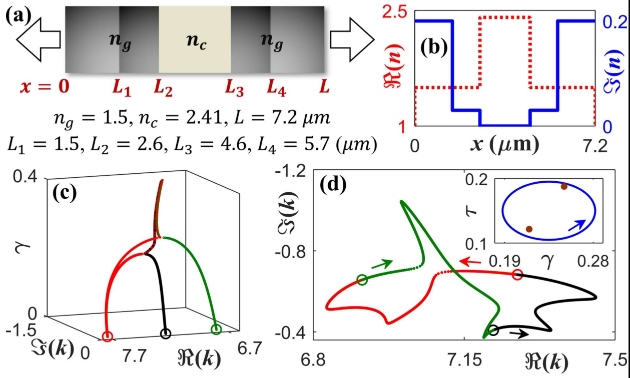

Fig. 1: **(a)** Schematic microcavity configuration **(b)** Complex profile of $n(x)$ for $\gamma = 0.2$ and $\tau = 0.15$ [dotted red and solid blue lines show the variations of $\Re(n)$ and $\Im(n)$, respectively]. **(c)** Coalescence of Pole-1 (red trajectory) with Pole-2 (black trajectory) and Pole-3 (green trajectory) for two different sets of γ and τ. **(d)** Successive adiabatic switching among three chosen poles in complex k-plane following an encirclement process in (γ, τ)-plane (inset; brown dots represent two EP2s). Three circular markers of respective colors indicate the initial positions of the three poles. Arrows indicate the direction of evolutions.

3. Conclusion: We have reported a successive switching scheme among three coupled cavity-states around an embedded EP3 with the simultaneous presence of two connected EP2s in an all-lossy Fabry-Pérot type optical microcavity which opens up a unique platform to realize low-noise device performance due to absence of external gain pumping.

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