

Estimation of Minimum Operating Voltage in FDSOI SRAM Using Gamma Distribution

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[Introduction] High minimum operating voltage ($V_{\min}$) of SRAM due to random variability has been a great problem. Previously, the extreme values of SRAM $V_{\min}$ are proved to follow Gumbel distribution [1]. In this work, the $V_{\min}$ distribution of all cells on a 32kb SRAM macro is measured, and its distribution characteristics are analyzed [2]. **[Measurement results]** SRAM device-matrix array test-element-group (DMA-TEG) fabricated by 65 nm FDSOI technology are used in this work [3-4]. The measurement method of $V_{\min}$ [5] is improved, and $V_{\min}$ of

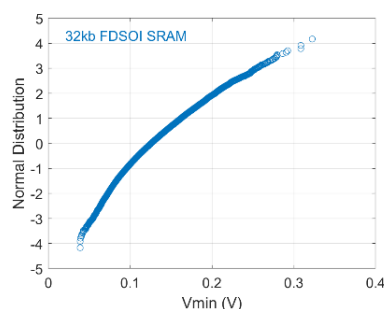


Fig.1. $V_{\min}$ distribution of 32kb FDSOI SRAM cells.

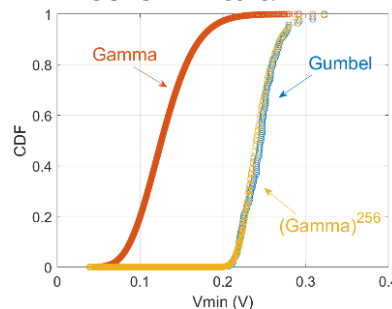


Fig.3. CDF of the gamma and Gumbel distribution.

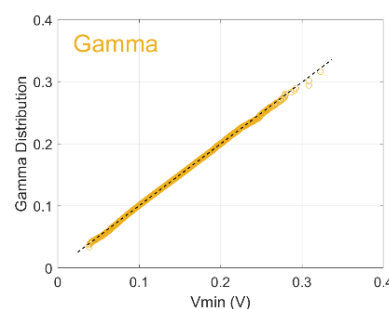


Fig.2. QQ-plot of fitted gamma distribution of $V_{\min}$.

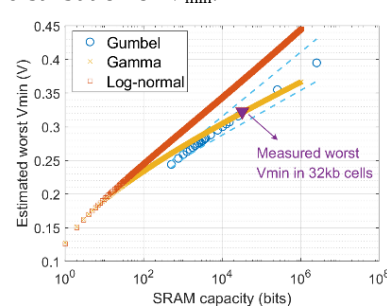


Fig.4. $V_{\min}$ estimation results of the three models.

32k cells are successfully measured. The distribution is shown in Fig. 1. The $V_{\min}$ values do not follow normal distribution. However, since $V_{\min}$ of all cells have non-negative values, they can be fitted with the Generalized Gamma (GENG) distribution, which includes exponential, Weibull, gamma and log-normal distribution as its special cases [6]. Further, after comparing all possible models, it is found that the gamma distribution fits best (Fig.2). According to the extreme value theory, for the measured $V_{\min}$ data, the cumulative distribution function (CDF) of the 256th power of fitted gamma distribution should be equal to the CDF of maximum values in every 256 cells, which is the Gumbel distribution [7]. Fig. 3 shows such consistency. Therefore, the gamma distribution of SRAM $V_{\min}$ is confirmed. This characteristic can be used for SRAM $V_{\min}$ estimation. By linear extrapolating from the fitted models, the SRAM worst $V_{\min}$ up to 1Mb cells is estimated (Fig. 4). The Gumbel model is fitted with 16kb cells, and its 95% confidence interval is included. The log-normal and gamma model use only 2kb cells within the measured 32kb as sample data. The gamma estimation results beyond 32kb fall in the Gumbel 95% confidence interval, and lies close to the measured worst $V_{\min}$, proving the correctness of the gamma model. **[Conclusion]** For 32kb FDSOI SRAM, $V_{\min}$ distribution of all cells are measured successfully. The gamma distribution of SRAM $V_{\min}$ is verified, and the $V_{\min}$ estimation by extrapolating from gamma distribution is proved feasible. **[Reference]** [1] T. Mizutani et al., JJAP. 53, 04EC18, 2014. [2] Yu et al. SSDM, p.59-60, 2021. [3] Y. Yamamoto et al., VLSI, p.109, 2012. [4] Y. Yamamoto et al., Symp. VLSI Tech. Dig., 2013, p. 212. [5] T. Mizutani et al., 2015 Jpn. J. Appl. Phys. 54 04DC16. [6] S. Yokogawa et al., 2019 IEEE International Reliability Physics Symposium (IRPS), 2019, pp. 1-6. [7] T. Mizutani et al., 2019 Silicon Nanoelectronics Workshop (SNW), 2019, pp. 1-2.