Extended Abstracts of the 17th Conference on Solid State Devices and Materials, Tokyo, 1985, pp. 9-12

A-0-3

Invited

Quantum Physics of 2D Electrons in Semiconductor Interfaces

S. Kawaji

Department of Physics, Gakushuin University,

Mejiro, Toshima-ku, Tokyo, 171, Japan

An overview of the recent progress performed in the last five years in the quantum mechanical transport properties in two-dimensional (2D) electron systems in Si-MOSFETs and GaAs/$Al_xGa_{1-x}As$ heterostructures is presented. It includes weak localization and negative magnetoresistance, quantum Hall effect and fractional quantum Hall effect.

§1 Introduction

Since 1979, a rapid progress has been made in understanding of quantum mechanical properties of electrical conduction in two-dimensional (2D) systems in semiconductor inversion layers such as in Si-MOSFETs and GaAs/$Al_xGa_{1-x}As$ heterostructures. Main topics are weak localization and negative magnetoresistance, quantum Hall effect and fractional quantum Hall effect.

In the present paper, fundamental concepts of these phenomena will be reviewed.

§2 Weak Localization and Negative Magnetoresistance

In 1958 Anderson[1)] discussed the electronic conduction in disordered systems and predicted that the quantum diffusion should be absent when the disorder is large enough. Based on many author's numerical studies of eigenstates in disordered systems and their system size dependences,[2)] Abrahams, Anderson, Licciardello and Ramakrishnan[3)] proposed a scaling relation between the conductance g(L) of the system and the system size L as

$$g(L) = f(g(L'), L/L') \tag{1}$$

where f is a universal function. They write eq.(1) in following form

$$\frac{d\,\ell n\, g(L)}{d\,\ell n\, L} = \beta(g(L)) \tag{2}$$

where $\beta(g(L)) = \{df(g(L),x)/d\,\ell n\, x|_{x=1}\}/g(L)$. They deduced the behaviour of $\beta(g)$ from a general consideration. In a macroscopic good conductor, we can define the conductivity $\sigma(L)$ in d-dimensions as $g(L) = \sigma(L)\, L^{d-2}$ and $\sigma(L)$ is independent of L. Therefore, we have an approximation of $\beta \simeq d - 2 - (g_a/g)$ near the metallic limit ($g \to \infty$). In 2D systems, we can derive the following result from $\beta(g) \simeq -g_a/g$ and eq.(1):

$$g(L) = g_0 - g_a\, \ell n(L/L_0) \tag{3}$$

where $g_a \ell n(L/L_0)/g_0 \ll 1$.

We have a result of perturbation calculation by Gorkow, Larkin and Khmelnizkii[4)] as

$$\sigma = \sigma_0 - \frac{2e^2}{\pi\hbar}\,\ell n\,\frac{L}{\ell} \tag{4}$$

where σ_0 is Drude conductivity $\sigma_0 = ne^2\tau/m$ and $\ell = v_F\tau$ is the electron mean free path.

The process of the perturbation calculation shows that the correction term to the Drude conductivity in eq.(4) comes from constructive interference between electron wave functions scattered by impurities such as $\Psi_{\vec{k}}$ and $\Psi_{-\vec{k}}$. This is the weak localization.

In actual 2D systems at finite temperatures, each electron is scattered inelastically with inelastic scattering time τ_ε and makes a transition from one localized state to another localized state. In other words, electron wave functions which construct a weakly localized state lose phase coherence after an inelastic scattering, then the electron is released from the localized state. Therefore, we have to replace L in eq.(4) by $L_\varepsilon = \sqrt{D\tau_\varepsilon}$ where $D = v_F{}^2\tau/2$ is the diffusion coefficient of electrons in two dimensions. Thus we have

$$\sigma = \sigma_0 - \frac{e^2}{\pi\hbar}\,\ell n\,\frac{\tau_\varepsilon}{\tau} \tag{5}$$

When we apply a magnetic field $\vec{B}$ perpendicular to the 2D systems, the phase in the electron wave function changes by $\int e\vec{A}\cdot d\vec{r}/\hbar$ where $\vec{A}$ is a vector

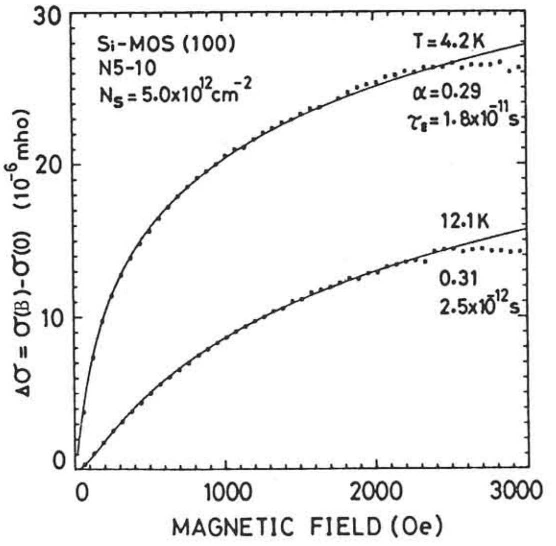

Fig. 1 Change in the conductivity $\Delta\sigma(B)$ versus magnetic field B in an n-channel (100) Si MOFET[5)]

potential; $\vec{B} = \nabla \times \vec{A}$. When we use the Landau gauge for the vector potential, the change in phase after an electron passes through a length R becomes $\Delta\phi = eBR^2/\hbar$. When $\Delta\phi$ becomes about unity the localization will be destroyed. For simplicity, we define a length $L_m = \sqrt{\hbar/eB}$. When we have $L_m < L_\varepsilon$, we have to replace L in eq.(4) by L_m. Then we have an approximate formula for negative magnetoresistance as

$$\sigma = \sigma_0 - \frac{e^2}{\pi\hbar^2}\ \ell n \frac{\hbar}{D\, eB\, \tau} \qquad (6)$$

Figure 1 shows experimental results of the change in conductivity against the magnetic field observed in an n-channel Si(100)-MOSFET[5)] explained by Hikami, Larkin and Nagaoka's theory.[6)] In Fig. 1, the theory is fitted to experimental data by adjusting τ_ε and α. α is a constant and depends on the intervalley scattering time.

2D electron system in GaAs/$Al_xGa_{1-x}As$ heterostructures has a spherical energy surface. Therefore, negative magnetoresistance data can be explained only parameter τ_ε.[7)] Kawabata's theory[8)] can explain experimental data[9)] in a magnetic field region higher than Hikami et al's theory. Kawabata's theory can also explain experimental data in Si-MOSFETs by adjusting τ_ε and the intervalley scattering time.[10)]

§3 Quantum Hall Effect[11)]

A classical expression for the Hall conductivity in 2D systems is given by

$$\sigma_{xy} = -\frac{N_s e}{B} + \frac{\sigma_{xx}}{\omega_c \tau} \qquad (7)$$

where N_s is the surface electron concentration, $\omega_c = eB/m^*$ and σ_{xx} is the diagonal conductivity. If the magnetic field is strong enough and the temperature is low enough to satisfy the condition $\omega_c\tau \gg 1$ and $\hbar\omega_c \gg k_BT$, there are well separated Landau levels whose center are described by $\varepsilon_{N'} = (N' + \frac{1}{2})\hbar\omega_c$ $(N' = 0,1,2,\cdots)$. Each Landau level can accomodate up to $1/2\pi\ell^2 = eB/h$ where ℓ is the radius of the ground Landau orbit. When $N_s = \nu eB/h$ (ν = integer) is fulfilled, we have $\sigma_{xx} = 0$ because no electron scattering exists in filled Landau levels. Then, the Hall conductivity is given by

$$\sigma_{xy} = -\nu\frac{e^2}{h}. \qquad (8)$$

In 1975 Ando, Matsumoto and Uemura[12)] calculated Hall conductivity of 2D systems in a strong magnetic field based on Kubo formula.[13)] They derived a very important conclusion on the Hall conductivity in a case when impurity bands are separated from mother Landau levels. They stated that eq.(8) is valid when $\sigma_{xx} = 0$ even if N_s is not equal to $\nu eB/h$. This condition is fulfilled in their theory when the Fermi level lies in gap regions among impurity bands and mother Landau levels for

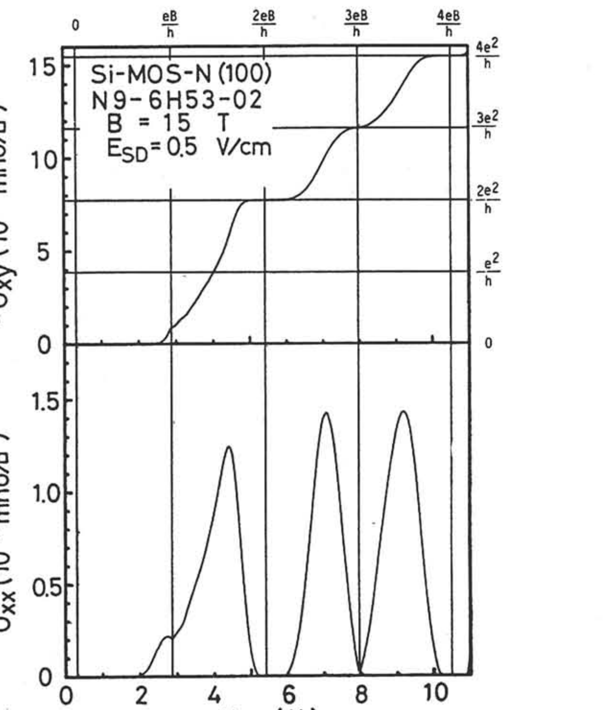

Fig.2 Hall conductivity σ_{xy} and diagonal conductivity σ_{xx} vs gate voltage V_G in the lowest four Landau levels[11,14)]. Reference 14 contains temperature dependence data.

$N_s \approx \nu eB/h$. We can extend their conclusion one step further to that eq.(8) is valid when $\sigma_{xx} = 0$ in case when the Fermi level lies in the localized states in lower and higher edges of any Landau level. Figure 2 shows experimental results obtained in 1980 by Kawaji and Wakabayashi.[14)]

In 1980 von Klitzing, Dorda and Pepper[15)] made a high precision measurement of Hall resistance $R_H = V_H/I$ and showed that $R_H = h/4e^2$ in accuracy of 3 ppm. They proposed that this is a new method for the determination of the fine structure constant $\alpha = \mu_0 c e^2/2h$. Since then the quantum Hall effect has attracted attention of many metrologists and physicists.

Laughlin[16)] derived eq.(8) by a general argument based on a thought experiment in a 2D looped ribbon in which only lossless Hall current is circulating as shown in Fig. 3. In this system, a vector potential A_1 along the ribbon can be changed slowly by the change in magnetic flux ϕ with no influence on B normal to surface on the ribbon. The change in ϕ or A_1 causes a change in the phase of the wave function $\Psi \rightarrow \Psi\exp(i2\pi\,\phi/\phi_0)$ where $\phi_0 = h/e$. For a localized state, the change in ϕ does not affect the energy. If the state is extended, however, after the change in ϕ by ϕ_0, the gauge invariance is satisfied and the system comes back to its original state. But during this process, the center coordinates of all extended electrons shift by $\Delta Y = 2\pi\ell^2/L$. This means that electrons are transfered from one Hall electrode to another and the energy of the systems changes by $\Delta U = \nu e V_H$ where ν is integer including zero. The current I is given by

$$I = -\frac{1}{L}\frac{\partial U}{\partial A_1} \qquad (9)$$

Here, ∂A_1 shoud be replaced by $\Delta A_1 = h/Le$ and $\Delta\partial U$ by $\Delta U = \nu e V_H$. Thus we have eq.(8).

Recent high precision measurements of quantized Hall resistance in silicon MOSFETs by Yoshihiro et al[17)] show that the Hall resistivity $\rho_{xy} = \frac{h}{\nu e^2}$ is verified to be constant, i.e., the value corresponding to h/e^2 is unchanged against electron concentration (for ν = 4, 8, 12), temperature (1.4 K $\sim$ 0.5 K), magnetic field strength (9.0 T $\sim$ 15 T), and channel current, to within one part in 10^7 when the measurement is made under the condition $\rho_{xx} < (4/\nu) \times 10^{-2}\ \Omega$.

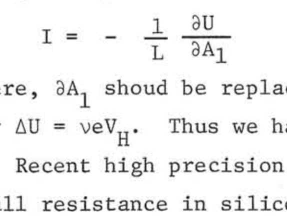

Fig.3 Laughlin's looped ribbon with circumference L. Magnetic flux ϕ changes the vector potental A_1 on the ribbon without changing the magnetic field B.

§4 Fractional Quatum Hall Effect

In 1982 Tsui, Störmer and Gossard[18)] measured ρ_{xx} and ρ_{xy} of GaAs/$Al_xGa_{1-x}As$ ($x \simeq 0.3$) heterostructure interfaces with $\mu = (8 \sim 10) \times 10^4\ cm^2/V\cdot s$ in the field up to 21 T at 0.5 K and found a plateau in ρ_{xy} at $\rho_{xy} = 3h/e^2$ and a dip in ρ_{xx}. Störmer et al[19)] observed in high mobility samples clear dips in ρ_{xx} at many fractional fillings of a Landau level. Some of them show clear plateaus in ρ_{xy}, too. The fractional filling factors are $\nu = q/p$ where p is an odd integer and q is an integer. Figure 4 shows our observations of the fractional quantum Hall effect.[20)]

Experimental results show that the fractional quantum Hall effect can be observed only in high mobility samples. This fact suggested that the Coulomb interaction between electrons play the most impotant role. Reviews on theoretical investigations are published.[21,22)] In the following, Laughlin's theory[23)] will be simply described.

When we use the Landau gauge for the vector potential in the free electron Hamiltonian, we have a solution described in a cylindrical coordinate system (r, ϕ). The wave function of the ground Landau level contains a factor $\exp(-|z|^2/$

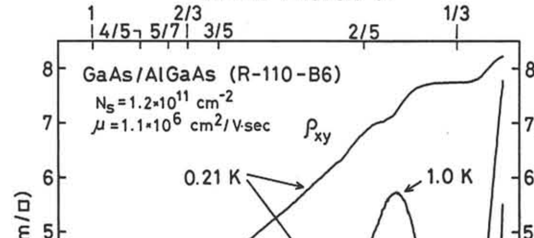

Fig.4 Magnetic field dependence of ρ_{xx} at T = 0.21 and 1.0 K and ρ_{xy} at 0.21 K in a GaAs/$Al_xGa_{1-x}As$ heterostructure with very high mobility[20)].

$4\ell^2)$ where $z = x + iy$. Laughlin constructed an approximate wave function of an interacting N electron system for $\nu = 1/p$ state $\Psi_{1/p}(z_1, z_2, \cdots z_N)$ as

$$\Psi_{1/p} = \prod_{j<k}^{N} (z_j - z_k)^p \exp\left(-\sum_{j=1}^{N} |z_j|^2/4\ell^2\right) \qquad (10)$$

The many body wave function must be antisymmetric for exchange of electrons. Thus p is an odd integer.

Laughlin found that the probability density $|\Psi_{1/p}(z_1, z_2, \cdots z_N)|^2$ calculated by eq.(10) has the same form as a Boltzmann factor of a classical one component 2D plasma consists of particles with charge $-p$ in a uniform positive background charge $1/2\pi\ell^2$ at temperature $T = p/k_B$. The charge neutrality requires particle density $n = 1/2\pi\ell^2 p$. Then, the filling factor is given by $\nu = 2\pi\ell^2 n = 1/p$. Borrowing calculated results of the classical plasma, Laughlin showed that the Coulomb energy of this state is smaller than the energy of the Hartree-Fock calculation. Therefore, the Laughlin state is the ground state separated by an energy gap from the excited state.

When the filling factor has a small deviation from 1/p, excess electrons or excess holes make quasiparticles with fractional charges of $\mp e/p$ other than the major Laughlin state. These quasiparticles can be easily trapped by disordered sites to form ρ_{xy} plateau against a change in the magnetic field.

§5 Concluding Remarks

2D electron systems in solid state device play very important roles in development of quantum physics in electrical conduction in solids. The negative magnetoresistance has made clear the quantum mechanical nature of electron localization and has developed a new tool to study microscopic electronic processes. The quantum Hall effect has shown the potential of a novel atomic standard of resistance. The fractional quantum Hall effect has shown a new electronic state where many body effect is essential. It is interesting to note here that disorders in material play an important role in these phenomena. Advancement of material technology in future will open fields in physics which we cannot imagine at present

A part of this work is supported by a Grant-in-Aids for Special Distinguished Research from the Ministry of Education, Science and Culture.

References

1) P. W. Anderson: Phys. Rev. 109(1958) 1492.
2) D. J.Thouless: *Ill-Condensed Matter*, ed. R. Balium, R. Maynord and G. Toulouse(North-Holland, Amsterdam, 1979) p.1
3) E. Abrahams, P. W. Anderson, D. C. Licciardello and T. V. Ramakrishnan: Phys. Rev. Lett. 42 (1979) 673.
4) L. P. Gorkow, A. I. Larkin and D. E. Khmelnizkii: Pisma ZH. Eksp. Theor. Fiz. 30 (1979) 248 [JEPT Lett. 30 (1979) 228].
5) Y. Kawaguchi and S. Kawaji: J. Phys. Soc. Jpn. 49 (1980) 1635.
6) S. Hikami, A. I. Larkin and Y. Nagaoka: Prog. Theor. Phys. 63 (1980) 607.
7) T. Nambu, S. Kawaji, K. Kuboki, Y. Kawaguchi, J. Yoshino and H. Sakaki: J. Phys. Soc. Jpn. 53 (1984) 682.
8) A. Kawabata: J. Phys. Soc. Jpn. 53 (1984)1429.
9) A. Kawaji, K. Kuboki, H. Shigeno, T. Nambu, J. Wakabayashi, J. Yoshino and H. Sakaki: Proc. 17th Int. Conf. Physics of Semicon., San Francisco, 1984, (Springer-Verlag, New York, 1985) p.413.
10) S. Kawaji and Y. Kawaguchi: J. Phys. Soc. Jpn. 53 (1984) 2868.
11) S. Kawaji: Proc. Int. Symp. Foundation of Quantum Mechanics, Tokyo, 1983, (Phys. Soc. Jpn., Tokyo, 1984) p. 327.
12) T. Ando, Y. Matsumoto and Y. Uemura: J. Phys. Soc. Jpn. 39 (1975) 279.
13) R. Kubo, S. J. Miyake and N. Hashitsume: Solid State Physics Vol. 17, ed. F. Seitz and D. Turnbull (Academic Press, New York, 1965) p. 269.
14) S. Kawaji and J. Wakabayashi: *Physics in High Magnetic Fields*, ed. S. Chikazumi and N. Miura (Springer, Berlin, 1981) p. 284.
15) K. von Klitzing, G. Dorda and M. Pepper: Phys. Rev. Lett. 45 (1980) 494.
16) R. B. Laughlin: Phys. Rev. B23 (1981) 5632.
17) K. Yoshihiro, J. Kinoshita, K. Inagaki, C. Yamanouchi, T. Endo, Y. Murayama, M. Koyanagi, J. Wakabayashi and S. Kawaji: Phys. Rev. to be published.
18) D. C. Tsui, H. L. Störmer, and A. C. Gossard: Phys. Rev. Lett. 22 (1982) 1559.
19) H. L. Störmer, A. Chang, D. C. Tsui, J. C. M. Hwang, A. C. Gossard and W. Wiegmann: Phys. Rev. Lett. 24 (1983) 1953.
20) S. Kawaji, J. Wakabayashi, J. Yoshino and H. Sakaki: J. Phys. Soc. Jpn. 53 (1984) 1915.
21) B. I. Harperin: Helv. Phys. Acto 56 (1983) 75.
22) D. Yoshioka: Prog. Theor. Suppl. 1985 to be published.
23) R. B. Laughlin: Surface Sci. 142 (1984) 3740.