# An Approximate Form of the "Lucky-Drift" Expression for Ionisation Coefficients Used in Evaluation of Superlattice Structures 

J.S. Marsland and R.C. Woods<br>Department of Electronic \& Electrical Engineering, Sheffield University<br>Mappin Street,<br>Sheffield, S1 3JD, U.K.


#### Abstract

An approximate form of the expression for the ionisation coefficients calculated using the "lucky drift" model is presented. This simplified expression enables analytic calculation of the ionisation coefficients in superlattice "staircase" avalanching structures using the approximation of a "hard" energy threshold for impact ionisation, and assuming that lucky-drifts start with a uniform probability distribution over all points in the superlattice period. The results for holes are compared with available Monte-Carlo simulations for a device made using the GaAs/GaAlAs system.


The "lucky-drift" model of impact ionisation in semiconductors has been developed by McKenzie and Burt (1) who give a closed form expression for the hole ionisation coefficient, in a bulk semiconductor material; for parabolic bands, and an energy-independent mean free path, this can be written in the form :

$$
\begin{equation*}
B=\frac{\lambda_{E} \exp \left(-\ell_{0} / \lambda_{E}\right)-\lambda \exp \left(-\ell_{0} / \lambda\right)}{\lambda_{E}^{2}\left(1-\exp \left(-\ell_{0} / \lambda_{E}\right)\right)-\lambda^{2}\left(1-\exp \left(-\ell_{0} / \lambda\right)\right)} \tag{1}
\end{equation*}
$$

where $\lambda_{E}$ is the mean free path for energy relaxing collisions, $\lambda$ is the hole's mean free path and $\ell_{0}$ is the distance taken by a ballistic hole to achieve the threshold energy $E_{T}$ (necessary for impact ionisation) - i.e. $\ell_{0}=E_{\mathrm{T}} / \mathrm{e} \varepsilon$ where $\varepsilon$ is the applied electric field. Under the same conditions, the following approximation is also valid :

$$
\begin{equation*}
\lambda_{E}=e \varepsilon \lambda^{2}(2 n+1) / 2 \hbar \omega \tag{2}
\end{equation*}
$$

where $\hbar \omega$ is the phonon energy and $n$ the BoseEinstein factor.

For most cases of practical interest it appears that the momentum-relaxation rate (involving phonons) is much larger than the energy-relaxation rate; this is equivalent to the inequality

$$
\begin{equation*}
\lambda_{E} \gg \lambda \tag{3}
\end{equation*}
$$

since $\lambda$ represents the carrier's mean free path between collisions with phonons. Using this fact, it is clear that an approximate form for $\beta$ is :

$$
\begin{equation*}
\beta \approx \frac{1}{\lambda_{E}\left(\exp \left(\ell_{0} / \lambda_{E}\right)-1\right)} \tag{4}
\end{equation*}
$$

For large values of applied electric field $\varepsilon$ this expression itself reduces to

$$
\begin{equation*}
\beta \approx 1 / l_{0} \tag{5}
\end{equation*}
$$

and for low values of $\varepsilon$, eqn. (4) becomes :

$$
\begin{equation*}
\beta \approx \exp \left(-\ell_{0} / \lambda_{E}\right) / \lambda_{E} \tag{6}
\end{equation*}
$$

as noted by McKenzie and Burt (1).
The values of $\beta \lambda$ calculated using eqn. (4) are compared to values calculated numerically using a Monte-Carlo technique (1) in Fig.1. The approximation is clearly justified, over a range of ( $\beta \lambda$ ) exceeding three decades.

The form of eqn.(4) is sufficiently simple to enable analytic evaluation of the ionisation coefficients in superlattice structures operating as avalanching devices. For example, consider a device with the valence band structure illustrated in Fig. 2 which, using the ${ }^{\text {GaAs/GaAlAs system, has }}$ been the subject of numerical investigation by Brennan (2). We make the simple assumption that lucky-drifts start with equal probability from all points in the period of the superlattice. In addition, in Fig.2, lucky-drifts starting from $\mathrm{d}<\mathrm{x}<\mathrm{L}$ can never reach the ionisation threshold $E_{T}$ because all the carrier's kinetic energy is destroyed at $x=L$ (assuming the carrier cannot tunnel through the hetero-barrier). Then eqn. (4)' becomes

$$
\begin{equation*}
\beta=\frac{1}{L} \int_{0}^{L} \frac{d x}{\lambda_{E}\left(\exp \left(\ell_{0}(x) / \lambda_{E}\right)-1\right)} \tag{7}
\end{equation*}
$$

and $\ell_{0}$ is now a function of the carrier's starting position at $x$. The carrier's potential energy at the start of its lucky-drift is

$$
\begin{equation*}
E=x\left(e \varepsilon L+\Delta E_{v}\right) / L \text { for } 0<x<L \tag{8}
\end{equation*}
$$

and lucky carriers will move until they have lost $E_{T}$ potential energy and thus their kinetic energy has reached $E_{T}$ (which is also a function of position, because the superlattice is graded).

Since the total energy of a lucky carrier remains constant, then

$$
\begin{equation*}
E=m e \varepsilon L+x^{\prime}\left(e \varepsilon L+E_{w}+\Delta E_{v}-E_{N}\right) / L-E_{w} \tag{9}
\end{equation*}
$$

where $m$ is the number of band discontinuities traversed during the lucky drift and the ionisation takes place at $\mathrm{x}=\mathrm{mL}+\mathrm{x}^{\prime}$. Solving eqns. (8) and (9) simultaneously gives
$\ell_{0}(x)=m L+x^{\prime}-x=\frac{L\left\{\left(E_{w}+\Delta E_{v}-E_{N}\right) m+E_{w}\right\}-x\left(E_{w}-E_{N}\right)}{e \varepsilon L+E_{w}+\Delta E_{v}-E_{N}}$
and eqn. (7) becomes

$$
\begin{align*}
\beta & =\left.\frac{1}{L} \int_{0}^{t} \frac{d x}{\lambda_{E}\left(\exp \left(\ell_{0} / \lambda_{E}\right)-1\right.}\right|_{m=n-1} \\
& +\left.\frac{1}{L} \int_{t}^{d} \frac{d x}{\lambda_{E}\left(\exp \left(\ell_{0} / \lambda_{E}\right)-1\right.}\right|_{m=n} \tag{11}
\end{align*}
$$

where $n$ is an integer such that

$$
\begin{equation*}
0<\left(n e \varepsilon L+\Delta E_{V}-E_{N}\right)<e \varepsilon L \tag{12}
\end{equation*}
$$

Eqns.(10) and (11) can now be manipulated to give the result

$$
\begin{align*}
& \beta=\frac{1}{\lambda_{E} L}\left\{\frac { 1 } { 2 J } \operatorname { l o g } _ { e } \left[\frac{\sinh \left\{\left(E_{w} n+(n-1)\left(\Delta E_{v}-E_{N}\right)\right) K+J t\right\}}{\sinh \left\{\left(E_{w} n+(n-1)\left(\Delta E_{v}-E_{N}\right)\right) K\right\}}\right.\right. \\
& \left.\left.X \frac{\sinh \left\{\left(E_{w}(n+1)+n\left(\Delta E_{v}-E_{N}\right)\right) K+J d\right\}}{\sinh \left\{\left(E_{w}(n+1)+n\left(\Delta E_{v}-E_{N}\right)\right) K+J t\right\}}\right]-\frac{d}{2}\right\} \tag{13}
\end{align*}
$$

where

$$
\begin{align*}
& J=\left(E_{N}-E_{w}\right) / 2\left(e \varepsilon L+E_{w}+\Delta E_{v}-E_{N}\right) \lambda_{E}  \tag{14}\\
& K=L / 2 \lambda_{E}\left(e \varepsilon L+E_{w}+\Delta E_{v}-E_{N}\right)  \tag{15}\\
& d=e \varepsilon L^{2} /\left(e \varepsilon L+\Delta E_{v}\right)  \tag{16}\\
& t=L\left(n e \varepsilon L+\Delta E_{v}-E_{N}\right) /\left(e \varepsilon L+\Delta E_{v}\right) . \tag{17}
\end{align*}
$$

The following parameters were used in order to give comparison with ref. (2):
$\lambda=30.2 \AA$; $\quad T=300 \mathrm{~K} ; ~ \hbar \omega=29 \mathrm{meV} ; \mathrm{E}_{\mathrm{w}}=2$ or 3 eV ; $\Delta \mathrm{E}_{\mathrm{v}}=0.14$ or $0.24 \mathrm{eV} ; \mathrm{E}_{\mathrm{N}}=1.424 \mathrm{eV}$.
$E_{N}$ and $E_{w}$ are the smallest and largest threshold energles, respectively, in the graded gap. The "lucky-drift" model used here makes the approximation of a "hard" threshold - i.e. a carrier immediately undergoes impact ionisation as soon as its kinetic energy reaches $E_{T}$. The more sophisticated "soft" threshold model may introduce some differences of detail (3), but Brennan (2) does not state whether a "hard" or "soft" threshold was assumed, nor the values used for $E_{N}$ and $E_{w}$. However, he does fix the effective field ${ }^{\mathrm{w}}$ for the electrons (i.e. including the effect of the sloping band edge) at $250 \mathrm{kV} \mathrm{cm}^{-1}$ and so for comparison purposes this convention is adopted here. This means that

$$
\begin{equation*}
F_{\mathrm{e}}=\varepsilon-\Delta \mathrm{E}_{\mathrm{c}} / \mathrm{eL}=250 \mathrm{kV} \mathrm{~cm}{ }^{-1} \tag{18}
\end{equation*}
$$

and

$$
\begin{equation*}
F_{h}=F_{e}+\left(\Delta E_{v}+\Delta E_{c}\right) / e L \tag{19}
\end{equation*}
$$

where $F_{e}$ and $F_{\text {are }}$ the effective fields for electrons and holes respectively. Note that $\mathrm{F}_{\mathrm{h}}$ depends on the sum and not the difference of $\Delta E_{v}^{h}$ and $\Delta E_{C}$, and that $F_{\text {must }}$ be used in place of $\varepsilon^{v}$ in eqn. (2). In eqn. (19), the value $\Delta E_{C}=0.56 \mathrm{eV}$ $-\Delta E_{v}$ was used in the present work.

The variation of $\beta$ with $L$ predicted by eqn. (13), with the above parameters, is shown in Fig.3, together with the results of Brennan (2). The two sets of results are not in agreement. Similar calculations for the electron ionisation coefficient show some agreement with the MonteCarlo results (4).

## References

(1) McKenzie, S., and Burt, M.G. : submitted to J. Phys. C.
(2) Brennan, K. : IEEE Trans. Elect. Dev. ED-32, 2197-2205 (1985)
(3) Marsland, J.S., Solid State Electronics (in press).
(4) Marsland, J.S., and Woods, R.C., in preparation.

## Acknowledgements

It is a pleasure to acknowledge many helpful discussions with Professor P.N. Robson (Sheffield), and Dr. M.G. Burt and Dr. M.J. Adams (British Telecom Research Laboratories, Ipswich).


Fig. 1 : Graph of $\beta \lambda$ against $E_{T} / e \varepsilon \lambda$. - : numerical results (1); ———: eqn. (4)


Total hole energy

Fig. 2 : Form of valence band edge and threshold energy for holes assumed in the text


Fig. 3 : Graph of $\beta$ vs. L for the structure in Fig. 2 evaluated as outlined in the text. Also shown are points due to Brennan (2); $+\Delta \mathrm{E}_{\mathrm{v}}=0.14 \mathrm{eV}$ and $*, \Delta \mathrm{E}_{\mathrm{v}}=0.24 \mathrm{eV}$.

