Signal and Noise Modeling for Striped Channel Field-Effect Transistors

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The cylindrical metal-semiconductor field-effect transistor (MESFET) has been studied for modeling the two-dimensional charge-control behaviors in striped channel field-effect transistors (FETs). Modifying the classical approach presented by van der Ziel and Pacel et al., the authors have developed the dc, small-signal, and noise model for cylindrical FETs. By using the present analysis, signal and noise properties for cylindrical FETs have been calculated and compared with the results for conventional FETs. Experimentally observed enhancement in charge-controllability is explained. Concerning noise properties, cylindrical FETs are shown to be equivalent to conventional FETs if transport properties are assumed to be identical. The striped channel FET is, however, considered to have a greater potential as an ultra low-noise amplifier for microwave and millimeter-wave frequencies due to the reduced short channel effects as well as superior transport properties for quasi-one-dimensional (Q1D) electrons.

I. Introduction

With the recent progress in lithography and crystal growth techniques, there have been considerable reports on examining electron transport properties in quasi-one-dimensional (Q1D) semiconductor wires. One of interesting approaches is the striped channel field-effect transistor (FET), which has parallel stripes of very narrow channels. Excellent dc and microwave performance for striped channel FETs has been demonstrated by Onda et al. and Kawasaki et al.. The striped channel FET has some attractive features:

1) Superior electron transports due to Q1D confinement;
2) improved charge-controllability; and
3) reduced short channel effects.

Transport properties for Q1D electrons have been studied by several authors. There have been, however, no adequate model for explaining operation of striped channel FETs. Two-dimensional (2D) charge control behaviors make it difficult to understand the operation of striped channel FETs. The aim of this paper is to propose a dc, small-signal, and noise model for striped channel FETs including the 2D charge control characteristics. According to Rensch's approach, the cylindrical metal-semiconductor FET (MESFET) will be studied for modeling striped channel FETs.

II. DC Conditions for Cylindrical FETs

Figure 1 illustrates geometries for the striped channel FET and cylindrical MESFET. The cylindrical MESFET consists of a cylindrical rod of conducting material with a radius a which is surrounded by a circular gate with a length Lg. Following Pacel et al., the channel is divided into two regions. Ohm's law applies for region I, while in region II, electrons travel with their saturation velocity u_s. According to the boundary conditions for Poisson's equation, cylindrical coordinates (r, φ, z) are introduced. The applied gate voltage V_g modulates the thickness of a depletion layer (z = b(x)) and hence the radius of an active channel b(x).

The gradual channel region (region I) for a cylindrical FET is analyzed based on the following assumptions:

1) The drift velocity v rises linearly with fields E_x (< E_y);
2) the longitudinal electric fields are negligible compared to the transverse fields (E_y), i.e., E_x << E_y; and
3) donors in the depletion layer are fully ionized.

Poisson's equation for the potential \( \Psi \) in region I is

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \Psi}{\partial r} \right) = -\frac{qN_d}{\epsilon} \quad \text{for} \quad b(x) \leq r \leq a \quad (1a)
\]

\[
\Psi_1(r, z) = V(x) \quad \text{for} \quad 0 \leq r \leq b(x) \quad (1b)
\]

\[
\Psi_2(\phi, z) = V_{gs} - V_h \quad (1c)
\]

\[
\frac{\partial \Psi_1}{\partial r} (a, z) = 0 \quad (1d)
\]

where \( N_d \) is donor concentration, \( \epsilon \) permittivity, \( V_{gs} \), the gate-source voltage, \( V_h \), the built-in voltage for the Schottky barrier.

![Fig. 1. Geometries for the striped channel FET.](image)
and \( V(x) \) the channel potential at a point \( x \). To simplify the analysis, normalized radii for the active channel are introduced.

\[
\frac{s}{a} = \frac{b(0)}{a}, \quad \frac{w}{a} = \frac{b(x)}{a}, \quad \frac{p}{a} = \frac{b(L_1)}{a}. \tag{2}
\]

From (1a)-(1d), \( V(x) \) is obtained in terms of \( w \).

\[
V(x) - V_{gs} + V_{th} = V_{po} \left( \frac{1 - \frac{w}{a}}{2} + \frac{w^2}{a^3} \ln a \right) \tag{3}\]

where \( V_{po} = qN_G e^2/(2e) \). From (3), the threshold voltage is

\[
V_{th} = V_{gs} - \frac{V_{po}}{2}. \tag{4}
\]

Starting with the usual equations, one obtains drain current, length of region I, and the voltage drop for region I as

\[
I_d = J_0 a^2, \tag{5}
\]

\[
L_1 = \frac{2V_{po} H(\epsilon)}{E_{kv} a^2} \tag{6}
\]

and

\[
\Delta V_2 = 2V_{po} H(\epsilon) \tag{7}
\]

respectively, where \( I_s = qN_G a^2 v_s \), and

\[
H(n) = \begin{cases} \frac{1}{n} \left( \frac{1}{n} - 1 \right) & \text{for } n = 1, 2, 3, \ldots \\ \frac{1}{2} (\ln n - n^2) & \text{for } n = 0. \end{cases} \tag{8}
\]

The potential for the velocity saturation region (region II) is expressed by the sum of a particular solution for Poisson's equation and a homogeneous solution \( \Psi_2(r, x) \). The Laplace's equation for \( \Psi_2 \) is

\[
\begin{align*}
\frac{\partial^2 \Psi_2}{\partial r^2} + \frac{\partial^2 \Psi_2}{\partial x^2} + \frac{1}{r} \frac{\partial \Psi_2}{\partial r} & = 0, \tag{9a} \\
\Psi_2(r, L_1) & = 0, \tag{9b} \\
\Psi_2(a, x) & = 0, \tag{9c} \\
\frac{\partial \Psi_2}{\partial x}(0, L_1) & = -E_s. \tag{9d}
\end{align*}
\]

The solution for above equations are approximated by

\[
\Psi_2(r, x) \approx \frac{aE_s}{p_0} \sinh \left( \frac{p_0 (x - L_1)}{a} \right) J_0(\frac{p_0 r}{a}) \tag{10}
\]

where \( J_0 \) is the Bessel function of order 0 and \( p_0 \approx 2.4048 \) is a minimum value for positive roots of \( J_0(\rho) = 0 \). Therefore, the channel voltage drop \( \Delta V_2 \) through region II is

\[
\Delta V_2 = \frac{aE_s}{p_0} \sinh \left( \frac{p_0 L_2}{a} \right). \tag{11}
\]

From comparison between (11) and the Grebene - Ghandhi formulation\(^2\), it is shown that the channel potential varies more rapidly in cylindrical FETs than in conventional FETs. Samming (7) and (11), one obtains the drain voltage \( V_{ds} \).

**III. Small-Signal Parameters for Cylindrical FETs**

From the analyses given in (13), we obtain the expressions for transconductance:

\[
g_m = 4 \pi e v_s J_f, \tag{12}
\]

drain output resistance:

\[
r_d = \frac{I_f}{4 \pi e v_s} \tag{13}
\]

and gate-source capacitance:

\[
C_{gs} = \frac{I_f}{\mu_0 e^2}(f_{sc} + f_{ac}), \tag{14}
\]

where

\[
J_f = \frac{1}{f_r} \left( \frac{2}{p_0^2} \cosh \left( \frac{p_0 L_2}{a} \right) - 1 \right), \tag{15}
\]

\[
f_r = 2 \ln p - \left( \ln \left( p + \frac{1}{4} \right) - \frac{1}{4} \ln \left( s - \frac{1}{4} \right) \right) \cosh \left( \frac{p_0 L_2}{a} \right), \tag{16}
\]

\[
f_{sc} = \frac{s^2}{p^2} \left( \frac{2}{p_0^2} \cosh \left( \frac{p_0 L_2}{a} \right) + p^2 s^2 \right), \tag{17a}
\]

\[
f_{ac} = \frac{s^2}{p^2} \left( \frac{2}{p_0^2} \cosh \left( \frac{p_0 L_2}{a} \right) - \ln p \right) \cosh \left( \frac{p_0 L_2}{a} \right) - 1. \tag{17b}
\]

According to the preceding analysis, striped (cylindrical) channel FETs and conventional GaAs FETs are analyzed. For both structures, identical transport parameters \( (\mu_0 = 4000 \text{cm}^2/\text{Vs} \) and \( v_s = 1.2 \times 10^6 \text{cm/s} \) are used. Figure 2 presents normalized transconductance \( (g_m/I_d) \) as a function of \( V_{gs} \) for cylindrical and conventional GaAs MESFETs. An enhancement in charge-controllability is expected for cylindrical FETs. This result qualitatively agrees with experimental findings reported by Onda et al.\(^4\). Figure 3 illustrates the unity current gain cutoff frequency \( (f_1) \) evolution versus \( I_d \) for a striped channel FET consisting of N cylindrical channels (radius \( a \)) and for a conventional GaAs FET with an equivalent channel width \( (Z_s = 2N a) \). Concerning \( f_1 \), these devices are equivalent as far as transport parameters are identical.
IV. Noise Source Model for Cylindrical FETs

The noise model for cylindrical FETs was developed by modifying the earlier work by van der Ziel\(14,15\), Staas-Haus-Pape\(11\) to a cylindrical FET version. The noise source is divided into two parts, i.e., Johnson noise from region I and high-field diffusion noise from region II. Only the final results will be written here. Details of analyses are given in 13.

The drain voltage fluctuation \(\Delta v_d^2\) caused by the Johnson noise is calculated based on the following assumptions\(11,14\):
1) Gradual channel approximation holds;
2) the elementary noise voltage at a small segment \((x_0 < x < x_0 + dx_0)\) in region I is described by the Nyquist formula:
\[
\Delta v_d^2 = \frac{4kT_\alpha \Delta f \Delta v_t}{I_d} \tag{18}
\]
where \(\Delta f\) is the frequency range, \(k\) the Boltzmann constant, and \(T_\alpha\) the effective noise temperature; and
3) the intervalley scattering effects are included based on a simple expression for \(T_\alpha\) presented by Baeztoit\(16\):
\[
T_\alpha = T_0\left(1 + \frac{P_0}{E_\mu^d}\right)T_0\left(1 + \frac{E_\mu^d}{\delta}\right) \tag{19}
\]
where \(T_0\) is the lattice temperature and \(\delta\) is an empirical parameter.

The drain voltage fluctuation \(\Delta v_d^2\) due to an elementary noise produced at \(x\) is obtained under ac-open-circuit drain conditions. Summing \(\Delta v_d^2\) over region I, \(\Delta v_d^2\) can be calculated as
\[
\Delta v_d^2 = \frac{8kT_\alpha \Delta f V_{ds}^2}{I_d} \cosh^2\left(\frac{\beta_0 I_d}{a}\right) [P_0 + P_3] \tag{20}
\]
with \(P_0 = \frac{1}{p^4} H(6)\) \((21a)\) and \(P_3 = \delta p^3 H(0)\). \((21b)\)

At high frequencies, there are induced noise currents on the gate which is capacitively coupled to the channel. The induced gate charge \(\Delta q_g\) due to an elementary noise produced at \(x_0\) is obtained with ac-short-circuit drain conditions\(11,18\). The induced gate currents can be derived by summing \(\Delta q_g^2\) over region I. The final expression can be written as
\[
\Delta q_g = \frac{8kT_\alpha \Delta f V_{ds}^2}{I_d} \left(\frac{2\alpha L_d}{v_{th}^d r_{th}}\right)^2 \cosh^2\left(\frac{\beta_0 I_d}{a}\right) [R_0 + R_d] \tag{22}
\]
with \(R_0 = \frac{1}{p^4} (\kappa^2 H(6) + \kappa^2 H(8) + \frac{\gamma^2}{4} H(10))\) \((23a)\) and \(R_d = \delta p^3 (\kappa^2 H(0) + \kappa^2 H(2) + \frac{\gamma^2}{4} H(4))\), \((23b)\) where
\[
\gamma = \frac{2\alpha L_d}{I_d} \cosh\left(\frac{\beta_0 I_d}{a}\right) \tag{24}
\]
and
\[
\kappa = \frac{1}{4H(6)} \frac{\pi^6 - \pi^6 - \frac{\pi^6}{2}}{24} (\ln s + \frac{1}{4})(25)
\]
where \(s = \frac{2\pi}{4H(6)} (26)\)

The diffusion noise from region II is interpreted as shot noise due to traveling dipole layers\(11\) generated at the rate
\[
\Delta v_d^2 = \frac{2D_n A \Delta f}{A} \tag{26}
\]
where \(A_d\) is the diffusion coefficient, \(n\) is the carrier density in the Stiator model, and \(D_n\) was assumed to be constant. However, the Monte-Carlo calculations indicate that \(D_n\) strongly depends on electric fields\(17\). Modifying the Einstein relation, we approximate the \(D_n\) variation versus \(E_d(\geq E_s)\) by

\[
D_n \approx D_b + \frac{kT_\alpha}{q} E_d \tag{27}
\]
where \(D_b\) is the high-field diffusion coefficient\(11\). Using \(27\), we calculate the drain voltage fluctuation \(\Delta v_d^2\) due to the diffusion noise. The final results are
\[
\Delta v_d^2 = \frac{2kT_\alpha \Delta f V_{ds}^2}{\pi^2 \alpha^d \gamma^d \eta^d \kappa^d (\beta_0 I_d)^2} [D + \bar{D}] \tag{28}
\]
with \(D = D_b(3 + 2\ln \lambda + \lambda^2 - 4\lambda)\), \((28a)\)

\[
\bar{D} = \frac{4kT_\alpha \mu_0}{\eta} (\lambda(\lambda - 2) - \lambda \ln \frac{2\pi^2}{1 + \lambda^2} - \frac{\lambda^2}{4} - \frac{1}{\lambda^2} - \frac{1}{\lambda^2} - \frac{1}{\lambda^2}) \tag{29b}
\]
where \(\lambda = \exp\left(\frac{\beta_0 I_d}{a}\right)\) and \(b_p = a_p\).

Modifying the Staas-Haus-Pape model\(11\), the diffusion noise induced gate currents are calculated, as is seen by
\[
\Delta i_{g} = \frac{8kT_\alpha \Delta f V_{ds}^2}{\pi^2 \alpha^d \gamma^d \eta^d \kappa^d (\beta_0 I_d)^2} [D + \bar{D}] \tag{30}
\]

The noise and drain noise currents are correlated since they originate both from the thermal noise in channel. Fluctuating fields and potentials modulate the carrier velocity as well as the depletion layer thickness. Since the velocity change gives no contribution to gate currents, the correlation coefficient \((C_{11})\) between \(i_{g2}\) and \(i_{g3}\) becomes so much smaller than unity.

\[
C_{11} = \frac{i_{g2}i_{g3}}{\sqrt{\Delta v_d^2} \Delta i_{g}} = \frac{S_0 + S_3}{\sqrt{[P_0 + P_3](R_0 + R_d)}} \tag{31}
\]
with \(S_0 = \frac{1}{p^4} \left(\frac{\gamma}{H(6)} + \frac{\gamma^2}{2} H(0)\right)\), \((32a)\) and \(S_3 = \delta p^3 \left(\frac{\gamma}{H(2)} + \frac{\gamma^2}{2} H(0)\right)\). \((32b)\)

Since the noise in region II causes no velocity change, there is a full correlation between \(i_{g2}\) and \(i_{g3}\). Therefore their correlation coefficient is

\[
C_{23} = 1. \tag{33}
\]

V. Noise Properties for Striped Channel FETs

The minimum noise figure \(F_{\min}\) is given in terms of small-signal parameters, drain noise coefficient \(P\), gate noise coefficient \(R\), and correlation coefficient \(C\), \((11,18)\), which are easily calculated according to the preceding analysis. Noise properties for striped channel FETs are studied and compared to the conventional GaAs FETs with an equivalent channel width.

Typical parameters used for calculations are \(V_{ds} = 0.8 V, \mu = 12.5, \delta = 0.4, \mu_0 = 4000 cm^2/Vs, v_s = 1.2 \times 10^7 cm/s, \) and \(D_b = 35 cm^2/s\). Identical parameters are assumed for both FETs.

Noise coefficients are calculated for striped channel and conventional FETs. Concerning \(P, R, C\) at optimum \(I_d\), no important superiority is expected for striped channel FETs. Figure 4 illustrates \(F_{\min}\) dependence on \(I_d\). The striped channel FET shows a comparatively flat dependence of \(F_{\min}\) on bias conditions. Regarding optimum \(F_{\min}\) values, these devices are equivalent. These results explain the experimental findings reported by Kawasaki et al. \(11\). The \(F_{\min}\) evolution versus frequency is shown in Fig. 5.

So far, striped channel FETs have been compared to conventional FETs with identical parameters. Figure 6 illustrates the influence of saturation velocity on \(F_{\min}\). It is shown that superior transports of Q1D channels dramatically improve noise characteristics for striped channel FETs.
Fig. 4. Evolution of $F_{\text{min}}$ (12GHz) versus $I_d$ for cylindrical (solid line) and conventional (dotted line) FETs with $R_g = R_c = 0$. Other device parameters are the same as those for Fig. 3.

Fig. 5. $F_{\text{min}}$ dependence on frequency for a striped channel FET (solid line) consisting of 10000 cylindrical channels and for a conventional FET (dotted line) with an equivalent gate width ($Z_g = 200\mu\text{m}$) ($L_g = 0.1\mu\text{m}$, $a = 100\AA$, $V_{\text{th}} = 2.1\text{V}$, and $R_c + R_g = 0$ and $\Omega$). * and X denote experimental data for striped and conventional HEMTs, respectively.

Fig. 6. Evolution of $F_{\text{min}}$ (12GHz) versus $I_d$ for a striped channel FET (solid line) consisting of 4000 cylindrical channels and for a conventional FET (dotted line) with an equivalent gate width ($Z_g = 200\mu\text{m}$) ($L_g = 0.28\mu\text{m}$, $a = 250\AA$, and $V_{\text{th}} = 2.1\text{V}$). The $v_s$ is ranged between 1.2 and $1.8 \times 10^{15}\text{cm/s}$. For a conventional FET, the result is shown only for $v_s = 1.2 \times 10^7\text{cm/s}$.

VI. Conclusion

In summary, the cylindrical MESFET has been studied for modeling the 2D charge-control behaviors in striped channel FETs. The authors have developed the dc, small-signal, and noise model for cylindrical FETs. By means of the present theory, signal and noise properties for striped channel FETs have been analyzed. Features for noise properties of striped channel FETs are

1) Supposing transport properties are identical, striped and conventional FETs are equivalent concerning optimum $F_{\text{min}}$;
2) a high $I_d$ due to superior transport properties of Q1D electrons improves the noise performance; and
3) suppressed short channel effects allow for reducing $L_g$ without suffering from an increase in the drain noise.

Consequently, the striped channel FET is considered to have a great potential to be a low-noise amplifier for microwave and millimeter-wave frequencies.

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