## Analytical Model for Circuit Simulation with Quarter Micron MOSFETs: Subthreshold Characteristics

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For deep submicron MOSFETs short-channel effects dominate the transistor characteristics. This is due to the increase of the lateral electric field. This paper provides a new simple model which includes the gradient of the lateral electric field in an analytical way. The model describes the subthreshold characteristics relating to short-channel effects correctly down to  $0.1 \mu m$  effective channel length  $L_{eff}$  with physical parameters  $(N_{sub}, C_{ox}, V_{fb}, \phi_f)$  taken from the long-channel device.

### 1. Introduction

For transistors with reduced channel lengths, the drain current  $I_D$  in the subthreshold region is no more independent of the drain voltage  $V_D$  but increases in absolute value. As a result, three important phenomena are observed.<sup>1)</sup> One is a reduction of the threshold voltage  $V_{th}$ . Second is a reduction of the body effect. Third is an increase of the subthreshold swing for increased  $V_D$ . Because of their many dimensional features, small geometry effects of MOSFETs have been investigated by two or even three dimensional numerical simulators.<sup>2,3)</sup> However, for circuit simulations a simplified analytical model is required. Unfortunately, the widely used chargesharing model<sup>4)</sup> is no longer appropriate for deep submicron MOSFETs.

By reducing the channel length but keeping applied voltages and other parameters as they are, only one physical quantity is changed, that is, the lateral electric field which becomes larger. The contribution of the lateral electric field causes the 2D current flow in the channel. Several authors have shown analytical solutions of the 2D Poisson equation.<sup>5,6</sup> However, either the solutions are too complicated for CAD applications or rather crude approximations are made. We show here that a simple analytical model can be given by combining experiment and theory basing on physical concepts.

## 2. Theory

A theory can be developed by applying the Gauss law to a narrow polygon in the depletion region under the gate. Under the assumption that the lateral electric field is independent of the vertical position, we get a simple relation between the vertical electric field at the surface  $E_x$  and the gradient of the lateral electric field  $E_y$ 

$$E_x(y) + \sqrt{B(\phi_s(y) - V_{sub})} E_{yy}(y) = q N_{sub} \sqrt{B\phi_s(y)} + Q,$$
(1)
$$B = 2\epsilon_{Si}/(q N_{sub}), \qquad E_{yy}(y) = dE_y(y)/dy$$

where  $N_{sub}$  is the substrate doping,  $\phi_s$  is the surface potential, and Q is the mobile charge density. At subthreshold Q can be neglected. Figure 1 shows the vertical field  $E_x$  at the surface calculated by the 2D simulator MINIMOS<sup>2</sup>) and by Eq. 1. In the analytical calculation the lateral field gradient is taken from the MINIMOS result. The reduction of the maximum  $E_x$  observed in the MINIMOS result can be reproduced by including the lateral electric field. The reduction is known as the drain-induced barrier lowering,<sup>7</sup>) which corresponds to the reduction of  $V_{th}$ for short-channel devices.

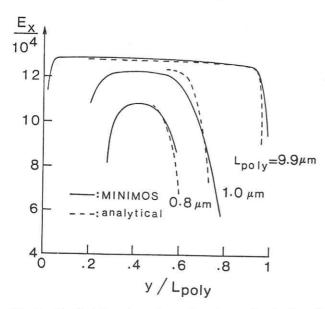


Fig.1.  $E_x$  distribution along the channel calculated by MINIMOS. Broken curves are results obtained by our theory. Applied voltages are the respective  $V_G = V_{th}$  for each  $L_{poly}$ ,  $V_D = 4V$ , and  $V_{sub} = 0$ .

#### 3. Results and discussion

## **3.1** The dependence of $\Delta V_{th}$ on $L_{eff}$

The threshold voltage shift from the longchannel device  $\Delta V_{th}$  is represented

$$\Delta V_{th} = A \sqrt{B\phi_s} E_{yy}, \qquad (2)$$
$$A = \epsilon_{Si}/C_{ox}, \ \phi_s = \phi_{s0} - V_{sub}$$

where  $\phi_{s0}$  is the surface potential at threshold for a long-channel transistor. The equation can be further simplified by giving an approximation that the surface potential is a linear function of  $V_G$  at threshold, resulting in

$$\Delta V_{th} \simeq \sqrt{c} A \sqrt{B} E_{yy} \quad \text{for} \quad E_{yy} \le 10^9 \frac{V}{cm^2}, \quad (3)$$

$$\Delta V_{th} \simeq a A^2 B E_{yy}^2 \qquad \text{for} \quad E_{yy} \gg 10^{10} \frac{V}{cm^2}.$$
 (4)

The parameters a and c define the gradient and the intersection of  $\phi_{s0}$  as a function of  $V_G$  around threshold, which can be given by solving the Poisson equation incorporating the contribution of  $E_y$  (cf. Eq. 1). Values for  $N_{sub} = 6 \times 10^{16} cm^{-3}$  are about 0.3 and 0.55V, respectively.

For an analytical model description of  $\Delta V_{th}$  we need an analytical expression for the lateral field gradient. We assume a quadratic function for the lateral

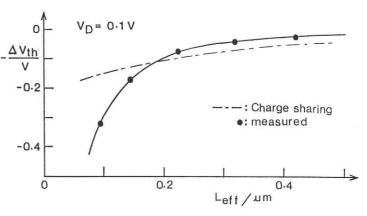


Fig.2. Calculated  $\Delta V_{th}$  at  $V_{sub} = 0$ . The result by the charge-sharing model is shown by a dotted broken curve.

potential distribution. Since the potential change is not drastic in the inversion channel, this approximation should be reasonable. Figure 2 shows calculated  $\Delta V_{th}$  values as a function of  $L_{eff}$  using this assumption. In the calculation no fitting parameter is used. The measured  $V_{th}$  for the long-channel transistor is used to determine the value of  $N_{sub}$ . The result of the charge-sharing model is depicted as well.

By comparing the calculated and measured  $\Delta V_{th}$  values at  $V_D \simeq 0$ , we can estimate  $L_{eff}$ . Figure 3 shows the result. For transistors studied here  $L_{eff}$  is aboutequal to  $L_{poly} - 0.5 \mu m$ . This is in good agreement with simulated 2D doping profiles. The  $L_{poly}$  length shorter than  $0.6 \mu m$  shows a saturation behavior due to the vicinity of two depletion regions from the source and the drain. For comparison measured  $\Delta V_{th}$  values are also plotted in Fig.2.

The MINIMOS result shows that the magnitude of  $E_{yy}$  for short-channel devices is around  $1 \times 10^9 V cm^{-2}$ . In this case Eq. 3 is a good approximation. Since the potential distribution is assumed to be a quadratic function, the dependence of  $\Delta V_{th}$ is  $L_{eff}^{-2}$ . If  $E_{yy}$  becomes larger than  $10^{10} V cm^{-2}$ , the dependence becomes  $L_{eff}^{-4}$ .

#### **3.2** The dependence of $\Delta V_{th}$ on $V_{sub}$

Figure 4 shows the  $V_{th}$  dependence on  $\sqrt{\phi_{s0} - V_{sub}}$  for different channel lengths. The agreement with measurements is very good. Thus the re-

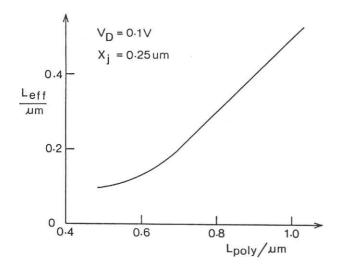


Fig.3. Calculated  $L_{eff}$  as a function of  $L_{poly}$ .

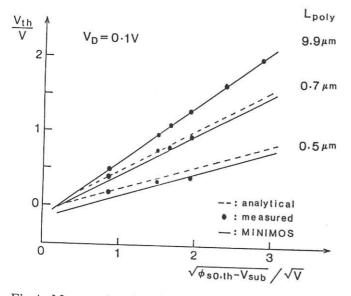


Fig.4. Measured and calculated  $V_{th}$  as a function of  $\sqrt{\phi_{s0} - V_{sub}}$ , where  $\phi_{s0}$  is the surface potential.

duction of the body coefficient for reduced channel lengths can be described correctly by the contribution of the lateral field gradient (cf. Eq. 2). A square root dependence on  $V_{sub}$  can be seen as well as in the MINIMOS result. This suggests that the lateral field gradient is independent of  $V_{sub}$ . On the other hand, the charge-sharing model shows a linear dependence of  $\Delta V_{th}$  on  $V_{sub}$ , which does not fit to the measurements.

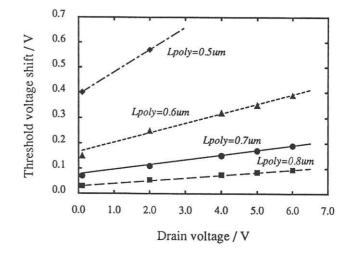


Fig.5. Measured  $\Delta V_{th}$  as a function of  $V_D$  at  $V_{sub} = 0$ .

# 3.3 The dependence of the subthreshold swing on $V_D$

The theory can be extended further to get an equation to evaluate the gate voltage swing S, which is given by

$$S/\log 10 = (1 + \frac{\gamma}{2\sqrt{\phi_s}} - \frac{A\sqrt{B}}{2\sqrt{\phi_s}} E_{yy} - A\sqrt{B\phi_s} \frac{dE_{yy}}{d\phi_{s0}})/\beta,$$
(5)

where  $\gamma$  is the body coefficient and  $\beta$  is the inverse of the thermal voltage.

As  $V_D$  increases, pinch-off occurs at the drain side. This is commonly treated as a reduction of  $L_{eff}$  by  $\Delta L$ . Since the potential difference within the inversion channel is small and can be assumed to be constant in the subthreshold region, the reduction of the channel length alone causes the increase of the lateral field gradient for increased  $V_D$ . This increase causes the increase of  $\Delta V_{th}$ . Thus, we can estimate  $\Delta L$  by fitting calculated  $\Delta V_{th}$  by Eq. 2 to measurements. Figure 5 shows the measured dependence of  $\Delta V_{th}$  on  $V_D$  for different channel lengths. Linear dependence with different gradients can be seen. This linear dependence of  $\Delta V_{th}$  on  $V_D$  can be written

$$\Delta V_{th}(V_D) = \Delta V_{th}(V_D \simeq 0) + gV_D. \tag{6}$$

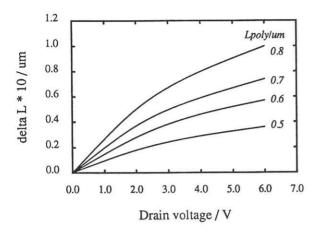


Fig.6. Calculated shortening of the channel length,  $\Delta L.$ 

The estimated value of g is again a linear function of  $\Delta V_{th}(V_D \simeq 0)$ . Because of this linearity the measured dependence of  $V_{th}$  on  $V_D$  for only one channel length is needed to get  $\Delta V_{th}(V_D)$  for all channel lengths. Figure 6 shows the calculated  $\Delta L$  values.

With increasing  $V_D$  values the subthreshold swing increases for short-channel transistors. This can be calculated with the  $\Delta L$  derived above. The result in Fig. 7 shows good agreement with measurement.

#### 4. Conclusion

Our model which includes the lateral electric field analytically can describe all subthreshold phenomena in a self-consistent way with physical parameters taken from the long-channel device. The potential distribution along the channel is approximated by a quadratic function, which seems suitable to reproduce measurements. Calculated reduction of the body coefficient and calculated increase of the gate voltage swing agree well with measurements with  $L_{eff}$  and its reduction as a function of  $V_D$  evaluated by combining the theory and the experiment.

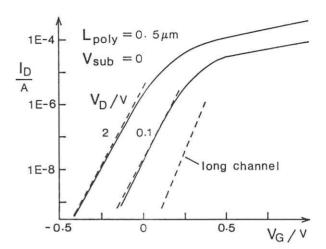


Fig.7. Subthreshold characteristics for the shortchannel device. Solid curves are measurements and broken lines are calculated gradients. The gradient of the long-channel device is shown for comparison.

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