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Can We Reduce Electron-Electron Scattering to Increase Electron Coherence Length and Reduce Noise in Quantum Wave Devices?

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Substantial effort in the area of quantum devices is devoted to the investigation of quantum interference devices. The present paper investigates performance limits on devices based on electron coherence. Below around 70 Kelvin electron–electron scattering is the most important scattering process, limiting the coherence length of electrons, and therefore the size and performance of quantum interference devices. The electron–electron scattering length has been calculated for quantum wells and quantum wires as a function of carrier density, well width, temperature, and electron excess energy above the Fermi level. It is shown that coherence lengths up to $100\mu m$ should be possible for typical conditions at liquid Helium temperatures and up to millimeters at more extreme conditions. In some respects, better performance is expected in quantum wires.

1 Introduction

Several types of quantum devices have recently been proposed—most are investigated at low temperatures and depend on the coherence of electron waves. It is therefore very important to have a clear picture of electron–electron scattering, which is the dominating scattering process at temperatures below about 70 Kelvin. Electron–electron scattering is responsible for phase breaking at low temperatures, it is the dominating process for carrier cooling in 2D and 3D devices, but it does not contribute directly to resistivity. It is well known from Fermi–Landau liquid theory that electrons have a finite lifetime, except exactly on the Fermi surface at zero temperature. Acoustic phonon scattering has been shown to freeze out [1], and turns out to be weaker than electron–electron scattering at low temperatures, which thus limits the size and performance of quantum interference devices, as confirmed in recent experiments [2].
The general properties of electron-electron scattering are well known [3] [4] [5]. A recent calculation [6] is based on an analytical calculation of [5]. In the present work, we calculate in detail the electron–electron scattering rates for GaAs based quantum wells and quantum wires as a function of temperature, carrier concentration, well width and electron excess energy above the Fermi level. We show ways to minimize the effect of electron–electron scattering to improve the design of quantum interference devices. It is shown that very long electron coherence lengths may be possible: for typical conditions and liquid Helium temperatures the coherence lengths are around $100\mu m$ while under more extreme conditions coherence lengths up to $1mm$ are predicted.

2 Scattering Rate Calculation

A typical electron–electron scattering process for a two dimensional electron gas (e.g in a HEMT structure or in a MOSFET) is shown in Fig. 1. Using time dependent perturbation theory, the total scattering rate for a test electron at wavevector $\mathbf{p}$ with spin σ is expressed as:

$$\left.\frac{1}{\tau_{ee}}\right|_{\mathbf{p}\sigma} = \frac{2\pi}{\hbar}\sum_{\mathbf{k},\mathbf{q},\sigma'} f_{\mathbf{k},\sigma'}\left(1-f_{\mathbf{k-q},\sigma'}\right)\left(1-f_{\mathbf{p+q},\sigma}\right)\times |V_{\mathbf{q}\sigma\sigma'}|^2\delta\left(E_{\mathbf{p+q},\sigma}+E_{\mathbf{k-q},\sigma'}-E_{\mathbf{p},\sigma}-E_{\mathbf{k},\sigma'}\right) \quad (1)$$

where $f_{\mathbf{k},\sigma'}\left(1-f_{\mathbf{k-q},\sigma'}\right)\left(1-f_{\mathbf{p+q},\sigma}\right)$ are the Fermi occupation factors and $V_{\mathbf{q}\sigma\sigma'}$ is the appropriate Coulomb interaction matrix element. The δ–function on the right expresses conservation of energy. The integral is over all electrons $\mathbf{k},\sigma'$ in the Fermi liquid and over all possible scattering vectors $\mathbf{q}$.
In two dimensions (2D) the bare Coulomb interaction matrix element is:

$$V_{\mathbf{q}} = \langle\mathbf{k-p},\mathbf{p+q}|V|\mathbf{k},\mathbf{p}\rangle = \frac{e^2}{2\epsilon_0\epsilon_r|\mathbf{q}|}F^{2D}_{ljki}(\mathbf{q}) \quad (2)$$

$F_{ljki}^{2D}(\mathbf{q})$ is the Coulomb Formfactor, which includes the shape of the wavefunctions. It is assumed that the wave functions are approximately given by:

$$\Psi(z) = \sqrt{2/w}\cos(\pi z/w) \tag{3}$$

In this case the Coulomb formfactor in 2D can be calculated analytically [7], while in 1D (quantum wire) it is done numerically. The 2D dielectric function $\epsilon(\mathbf{q}, (E_{\mathbf{p}} - E_{\mathbf{p+q}})/\hbar)$ enters the scattering rate as the screening factor.

The scattering rate is calculated by integrating Eq. 1 in the following way: We consider a test electron at wave vector $\mathbf{p}$ with spin σ. It can be easily seen that for any scattering partner at wave vector $\mathbf{k}$, all possible final scattering states $\mathbf{p}+\mathbf{q}$ and $\mathbf{k}-\mathbf{q}$ lie on a circle through the points $\mathbf{k}$ and $\mathbf{p}$. The numerical integration is performed in two steps: first a two-dimensional integral over θ and ϕ and finally the integration over the abolute value of k, as shown in Fig. 1.

For the quantum wire calculation very similar techniques are used [9]. An important difference is that the Coulomb form factor, which introduces the wire

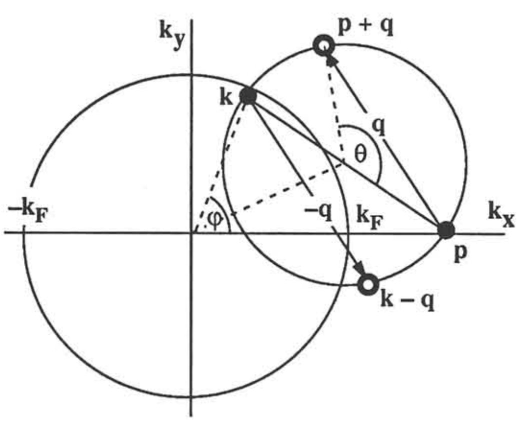

Figure 1: Schematic of electron-electron scattering process. Such processes are dominating for phase breaking in low temperature quantum interference devices and for carrier thermalization in 2D and 3D. In 2D all final states for scattering of an electron at $\mathbf{p}$ with an electron at $\mathbf{k}$ lie on a circle as shown in the figure.

or well shape into the calculation, has very different form in 1D and 2D. While the Coulomb formfactor for 1D diverges as $k \to 0$, it only has a weak dependance on k in 2D with a finite value at $k = 0$.

3 Electron-Electron Scattering in Quantum Wells

Fig. 2 shows the calculated electron–electron scattering lengths. The electron–electron scattering rates saturate when the temperature T falls below $T < \Delta/k_B$, i.e. when the temperature broadening of the phase space for scattering becomes small compared to it's width. The obtained scattering lengths are remarkably long. Typical conditions show coherence lengths of around $100\mu m$, while for electrons close to the Fermi surface, and for milli Kelvin temperatures, coherence lengths in excess of millimeters are predicted, provided other scattering mechanisms do not mask these effects.

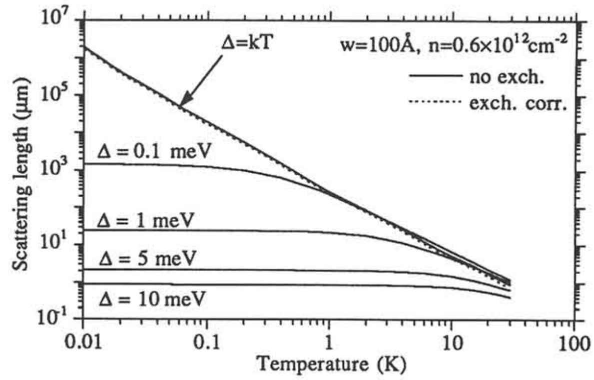

Figure 2: Electron–electron scattering length in a 2D quantum well as a function of temperature for electrons with different excess energies Δ above the Fermi surface. Curve labelled $\Delta = kT$ shows calculation for electron with variable excess energy simulating transport conditions close to thermal equilibrium.

4 Electron-Electron Scattering in Quantum Wires

Fig. 3 shows the calculated electron–electron scattering lengths as a function of temperature for different excess electron energies Δ above the Fermi level. The results are shown both for a calculation including (dashed curves) and excluding (solid curves) exchange and correlation terms.

The most striking result is, that the electron scattering lengths do not saturate as a function of temperature as they do in 2D. The absence of saturation at low temperatures is a consequence of the dramatic difference in the scattering phase space geometries between 2D and 1D. Due to the different phase space geometries, the dependence of scattering length on Δ is much weaker in 1D than in 2D. Fig. 4 shows the dependence of the electron-electron scattering length on the quantum wire width. This dependence originates from the dependence of the Coulomb formfactor on the wire width. The Coulomb form factor enters the calculation both in the unscreened Coulomb matrix element itself, as well as in the screening factor. The electron–electron scattering length increases as

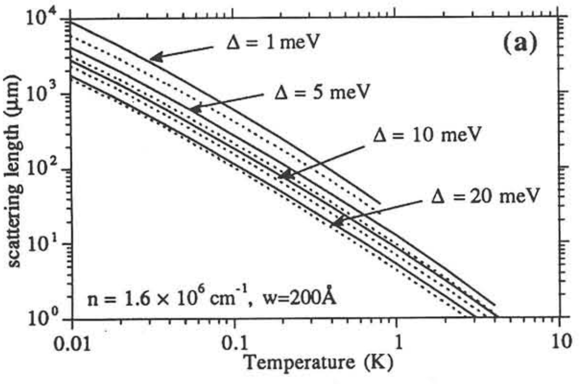

Figure 3: Electron–electron scattering length in a 1D quantum wire as a function of temperature for electrons with different excess energies Δ above the Fermi surface. Dashed line shows calculation including exchange—correlation effects, while solid lines are without exchange—correlation effects.

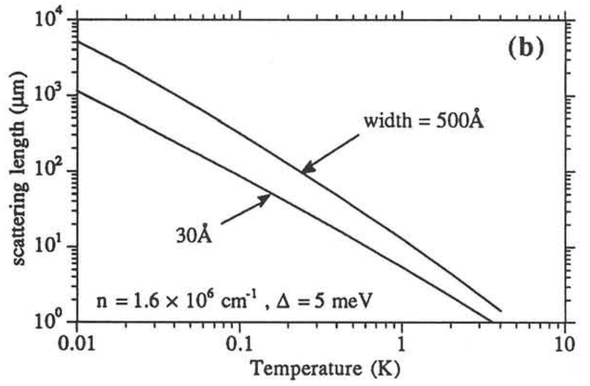

Figure 4: Electron–electron scattering length in a 1D quantum wire as a function of temperature for different quantum wire widths.

the well width is increased because of the weaker wave function overlap. Thus a minimum in the scattering rate is expected, since the scattering rate will increase again for wider wires, when intersubband scattering will start to set in.

5 Summary

At low temperatures electron–electron scattering limits the size and performance of proposed electron interference devices and it contributes to electron cooling. We have calculated electron–electron scattering rates and scattering lengths for the 2D and 1D electron gas of heterojunctions and quantum wells as a function of electron density, temperature, well width and electron excess energy. We show that spectacularly long electron coherence lengths are predicted for extreme conditions. Coherence lengths of up to $100 \mu m$ are calculated for typical conditions at liquid Helium temperatures, and coherence lengths up to the millimeter range may be possible under more extreme conditions. We discuss several parameters, which influence the strength of electron–electron scattering and which may be used to optimize the coherence length for quantum interference devices.

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