

## Novel Impact Ionization Model for Device Simulation Using Generalized Moment Conservation Equations

Ken-ichiro SONODA\*, Mitsuru YAMAJI, Kenji TANIGUCHI, and Chihiro HAMAGUCHI

*Department of Electronic Engineering, Osaka University, Suita, Osaka, 565 Japan*

Impact ionization (I.I.) phenomenon in inhomogeneous electric field is modeled using second- and fourth-order moments of a distribution function, which are average energy and average square energy, respectively. The model predicts the I.I. coefficient in both increasing and decreasing electric field regions, which appear in the vicinity of the drain of MOSFET's, better than the conventional field- or energy-dependent model. A conservation equation for the fourth-order moment is derived from the Boltzmann transport equation (BTE), which is combined with hydrodynamic model equations to give generalized moment equations. These equations are solved to use the new I.I. model in practical device simulation. The validity of the new I.I. model, incorporated with the generalized moment conservation equations, is certified by comparing the calculated generation rate in an  $n^+nn^+$  structure with that from Monte Carlo simulation, in which the BTE and the Poisson equation are solved self-consistently. The new I.I. model is also applied to the calculation of I.I. generation rate in an  $n$ -channel MOSFET.

### 1. INTRODUCTION

Device degradation caused by hot carriers has been main concern from the reliability point of view. Because secondary-generated carriers created by impact ionization (I.I.) have great influence on the degradation of gate oxide, accurate modeling of I.I. is necessary.

As a probability of an I.I. process depends on kinetic energy of a carrier, an I.I. coefficient, which means the number of ionization events per unit length, is determined by energy distribution function of carriers. The I.I. coefficient has been conventionally expressed as a function of electric field<sup>1)</sup>. In homogeneous electric field, it is reasonable to use the electric field to calculate the coefficient because there exists one-to-one relationship between electric field and the shape of distribution function.

In spatially varying electric field, which is the case in real devices, the energy distribution does not reach the state which corresponds to the local electric field, and the ionization coefficient is no longer expressed by the local electric field alone. In this case, it is more appropriate to use average energy<sup>2)</sup> rather than electric field to formulate impact ionization phenomena.

In the past few years, it has been pointed out<sup>3,4)</sup> that the average energy is still insufficient to describe the nonlocal nature of the I.I. phenomenon in strongly non-uniform electric field, which is commonly appears in modern scaled-down devices.

We propose an I.I. model which is formulated using second- and fourth-order moments of distribution function for precise description of I.I. in inhomogeneous electric field. The second- and fourth-order moments are average energy and average square energy, respectively. A set of model equations for carrier transport is also presented to perform practical device simulation with the I.I. model.

### 2. IMPACT IONIZATION MODEL

For the purpose of the investigation of the I.I. phenomena in inhomogeneous field, we use the Monte Carlo (MC) simulation program with analytical multi-valley band structure, in which scattering rates<sup>5)</sup> and the impact ionization rate<sup>6)</sup> as a function of electron energy are implemented.

Calculated average energy,  $\langle\epsilon\rangle$ , and impact ionization coefficient,  $\alpha$ , in the inhomogeneous electric field (Fig. 1(a)) are shown in Figs. 1(b) and (d), respectively ( $\langle A \rangle$  means  $\int A f d\mathbf{k} / \int f d\mathbf{k}$  hereafter.). These figures show that the impact ionization coefficient is uniquely determined from neither electric field nor average energy in spatially varying electric field.

Energy distribution functions in the field increasing or decreasing regions are plotted in Fig. 2. In the field increasing region (point A in Fig. 1), the distribution function shows steep decline in the high energy range. On the other hand, the distribution function at point B, where the field is decreasing, extends to higher energy range in spite of the same average energy at point A. The different shape of the distribution functions make it difficult to express the ionization coefficient as a function of average energy only.

In order to evaluate impact ionization more precisely, we use a fourth-order moment of the distribution function  $\langle\epsilon^2\rangle$ , in addition to second-order moment  $\langle\epsilon\rangle$ . The fourth-order moment is parameterized in a normalized form,  $\xi = \sqrt{(3/5)\langle\epsilon^2\rangle/\langle\epsilon\rangle}$ . The factor  $\sqrt{3/5}$  is introduced so as to  $\xi = 1$  when the distribution function is Maxwellian. Figure 1(c) shows the parameter,  $\xi$ , calculated using MC simulation. The figure shows that  $\xi$  starts to increase where the field decreases, which means that the high energy tail of the distribution function remains in spite of the rapid decrease of electric field.

Figure 3 shows calculated ionization coefficient for several field profiles as a function of the inverse of the average energy with several  $\xi$ 's as a parameter. The figure shows that ionization coefficient,  $\alpha$ , is expressed as

\*On leave from ULSI Laboratory, Mitsubishi Electric Corporation.

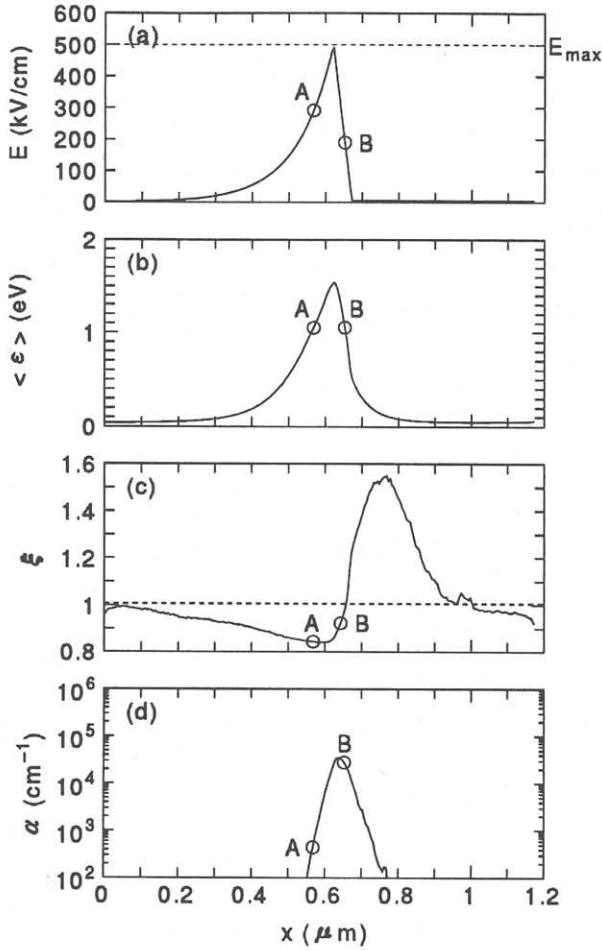


Fig. 1 Results of Monte Carlo simulation at a given electric field profile. (a) Electric field (increases exponentially and decreases linearly), (b) Average energy, (c)  $\xi \equiv \sqrt{(3/5)\langle\epsilon^2\rangle/\langle\epsilon\rangle}$ , (d) Impact ionization coefficient.

$\alpha_0 \exp(-\epsilon_c/\langle\epsilon\rangle)$  at a given  $\xi$ , where  $\alpha_0$  is constant and  $\epsilon_c$  depends on  $\xi$ . A slope of the  $1/\langle\epsilon\rangle$ - $\log \alpha$  plot is shown in Fig. 4 as a function of the parameter,  $\xi$ . From this figure, the impact ionization coefficient,  $\alpha$ , is modeled as

$$\alpha = \alpha_0 \exp\left(-\frac{\epsilon_{c0} \exp(-\gamma\xi)}{\langle\epsilon\rangle}\right). \quad (1)$$

This model equation, depending on the parameter  $\xi$ , gives different value of  $\alpha$  even at same average energy.

### 3. MOMENT CONSERVATION EQUATIONS

Both second-order moment,  $\langle\epsilon\rangle$ , and fourth-order one,  $\langle\epsilon^2\rangle$ , are required to use the new I.I. model (1) in device simulation. The fourth-order moment is numerically calculated from the conservation equation for  $\langle\epsilon^2\rangle$  incorporated in the hydrodynamic model<sup>7,8)</sup>. The conservation equations derived from the Boltzmann transport equation (BTE) are

$$\nabla \cdot (n\langle u\epsilon^2 \rangle) = -2qE \cdot S - n \frac{\langle\epsilon^2\rangle - \langle\epsilon^2\rangle_0}{\tau_{\langle\epsilon^2\rangle}} - U_{\langle\epsilon^2\rangle} \quad (2)$$

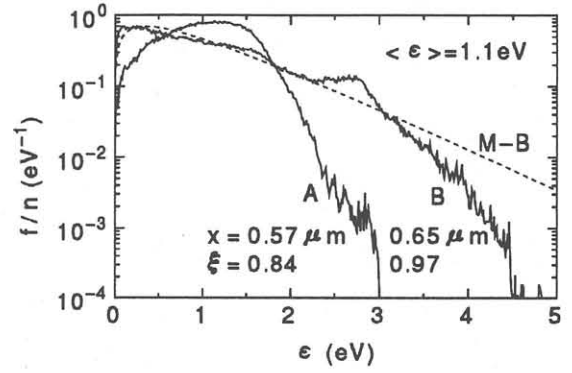


Fig. 2 Energy distribution functions at points A and B in Fig. 1 where the average energy  $\langle\epsilon\rangle = 1.1$  eV. The dotted curve means Maxwell-Boltzmann distribution function for the same average energy.

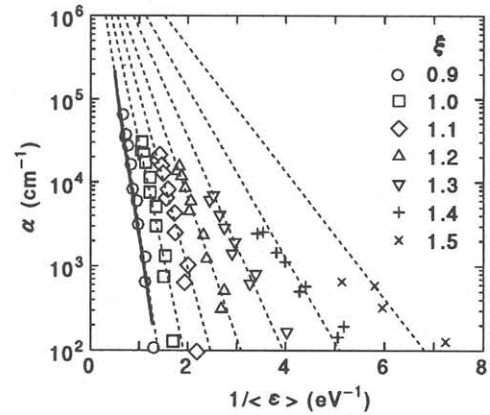


Fig. 3 Impact ionization coefficient as a function of inverse average energy for different  $\xi$  values. Dashed lines are obtained from least square fit to the data. Solid line indicates the ionization coefficient in homogeneous electric field.

$$n\langle u\epsilon^2 \rangle = \frac{\tau_{\langle u\epsilon^2 \rangle}}{\tau_{\langle u \rangle}} \frac{7}{3} \left( \langle\epsilon^2\rangle \frac{J}{-q} - \frac{kT_n}{q} n\mu \nabla \langle\epsilon^2\rangle \right), \quad (3)$$

where  $u$  is the group velocity of an electron,  $E$  is the electric field,  $J \equiv -qn\langle u \rangle$  is the electric current density,  $S \equiv n\langle u\epsilon \rangle$  is the energy flux,  $\langle\epsilon^2\rangle_0$  is the fourth-order moment at thermal equilibrium,  $U_{\langle\epsilon^2\rangle}$  is the net loss rate of  $\langle\epsilon^2\rangle$  due to generation-recombination process,  $\tau_{\langle A \rangle}$  is the relaxation time of  $\langle A \rangle$ ,  $\mu$  is the electron mobility, and  $T_n$  is the electron temperature defined by  $3kT_n/2 \equiv \langle\epsilon\rangle$ . The parameters,  $\tau_{\langle\epsilon^2\rangle} = 0.29$  ps,  $\tau_{\langle u\epsilon^2 \rangle}/\tau_{\langle u \rangle} = 0.59$  are extracted from MC simulation in homogeneous electric field. These parameters are almost independent on electric field or average energy.

### 4. RESULTS AND DISCUSSIONS

In order to verify the new I.I. model, the I.I. generation rate,  $G_{II}$ , in an  $n^+nn^+$  structure is calculated using the moment conservation equations with different I.I. models. They are compared with MC result in Figs. 5(a) and (b). Parameters used in the I.I. models are calibrated to provide the same I.I. coefficient

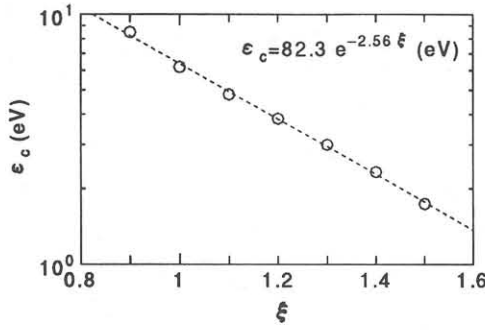


Fig. 4 The slope of the data in Fig. 3,  $\epsilon_c$ , as a function of the parameter,  $\xi$ . Dashed line and the formula in the figure indicate the least square fit to the data.

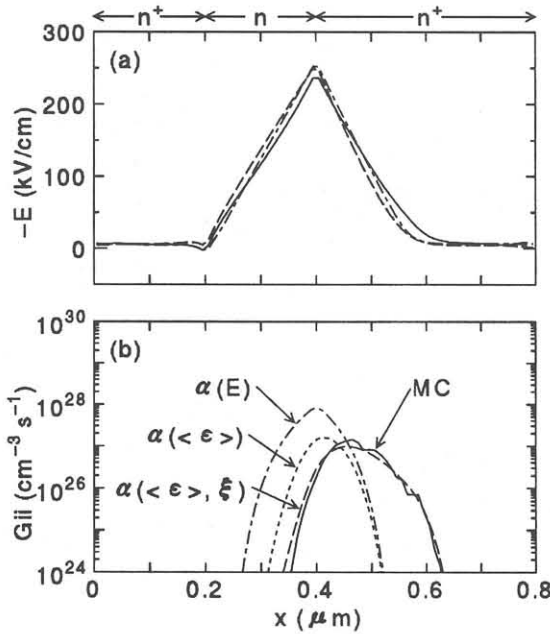


Fig. 5 Calculated electric field and impact ionization rate in an  $n^+nn^+$  structure ( $n^+ = 2 \times 10^{17} \text{ cm}^{-3}$ ,  $n = 5 \times 10^{15} \text{ cm}^{-3}$ ). Applied voltage is 5V. (a) Electric field, (b) Impact ionization generation rate.

obtained from MC simulation in homogeneous electric field. In the MC simulation, the BTE and the Poisson equation are solved self-consistently.

The calculated results with the local I.I. coefficient,  $\alpha(E)$ , overestimate maximum I.I. rate nearly one order of magnitude. Moreover, it underestimates the generation rate in the field decreasing region. Although the maximum I.I. rate is improved with  $\alpha(\langle \epsilon \rangle)$ , in which  $\alpha$  is expressed as a function of average energy only, the model still underestimates  $G_{II}$  in the field decreasing region. In contrast with the previous two models, the new model ( $\alpha(\langle \epsilon \rangle, \xi)$ ), provides the correct generation rate, which agrees with the MC result over the whole region.

The moment conservation equations are also applicable to two- or three-dimensional problems. The calculated I.I. generation rate in an  $n$ -channel MOSFET is shown in Fig. 6 and compared with the result of a con-

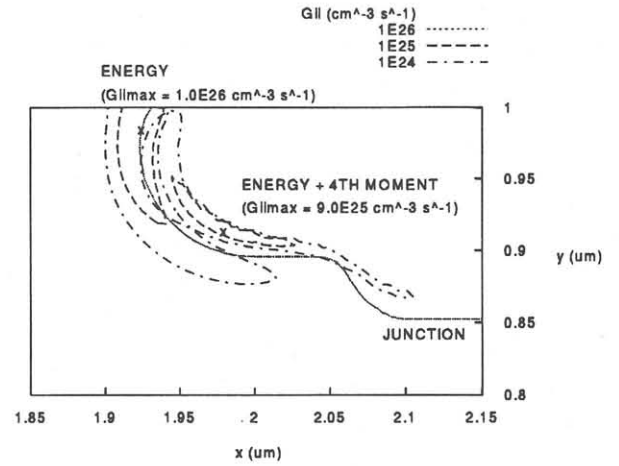


Fig. 6 Impact ionization generation rate at the drain junction of an  $n$ -channel MOSFET at  $V_d = 3V$ , and  $V_g = 1.4V$  with two I.I. models. The mark 'x' indicates the point of the maximum generation rate. The gate length is  $1 \mu\text{m}$  ( $1 \leq x \leq 2 \mu\text{m}$ ). The Si-SiO<sub>2</sub> interface lies at  $y = 1.0 \mu\text{m}$ .

ventional (average energy dependent) model. In spite of the same order of magnitude of the calculated substrate current for these two models, the new model suggests I.I. events occur frequently farther away from the Si-SiO<sub>2</sub> interface.

## 5. CONCLUSION

We proposed the I.I. model using second- and fourth-order moments of the distribution function, which is applicable for inhomogeneous electric field. The model combined with the generalized moment conservation equations makes it possible to predict the I.I. generation rate precisely in practical device simulation. The validity of the new model with the conservation equations was verified through the comparison with MC simulation in the  $n^+nn^+$  structure.

## References

- 1) A. G. Chynoweth: *Phys. Rev.* **109** (1958) 1537.
- 2) M. Fukuma and W. W. Lui: *IEEE Electron Device Lett.* **EDL-8** (1987) 214.
- 3) J.-G. Ahn, Y.-J. Park and H.-S. Min: *1993 VPAD* (1993) 28.
- 4) P. Scrobahaci, and T.-W. Tang: *IEICE Trans. Electron.* **E77-C** (1994) 134.
- 5) T. Kunikiyo, M. Takenaka, Y. Kamakura, M. Yamaji, H. Mizuno, M. Morifuji, K. Taniguchi, and C. Hamaguchi: *J. Appl. Phys.* **75** (1994) 297.
- 6) Y. Kamakura, H. Mizuno, M. Yamaji, M. Morifuji, K. Taniguchi, C. Hamaguchi, T. Kunikiyo and M. Takenaka: *J. Appl. Phys.* **75** (1994) 3500.
- 7) K. Bløtekjær: *IEEE Trans. Electron Devices* **ED-17** (1970) 38.
- 8) R. Thoma, A. Emunds, B. Meinerzhagen, H. Peifer and W. L. Engl: *IEDM Tech. Dig.* (1989) 139.