Extended Abstracts of the 1997 International Conference on Solid State Devices and Materials, Hamamatsu, 1997, pp. 306-307

Invited

Quantum Hopfield Network Using Single-Electron Circuits

Masamichi Akazawa
Faculty of Engineering, Hokkaido University, Sapporo 060, Japan
Phone: +81-11-706-7149, Fax: +81-11-706-7890,
E-mail: akazawa@sapiens.huee.hokudai.ac.jp

1. Introduction

This paper proposes the concept of the *quantum Hopfield network,* a quantum version of the classical Hopfield network. This concept provides an efficient way of solving combinatorial problems, including nondeterministic polynomial-time complete (NP-complete) problems, which are difficult for conventional computation models.

Introducing quantum mechanics into computation may produce the capability for massive parallel processing. The quantum generalization of the Turing machine, known as the quantum Turing machine,[1)] is an example. The quantum Turing machine can perform ultrahigh-speed computation because it can accept as input a coherent superposition of many different data and subsequently perform a computation on all of these input data simultaneously. This parallelism can be used to quickly solve several problems that are difficult with the classical Turing machine, such as *factoring* and *discrete logarithms*.

Is this type of quantum effect exclusive to the Turing machine? The author does not think so. Various other computation models besides the Turing machine are known, and it is likely that the parallelism of each of them can be enhanced with the application of quantum mechanics. This paper takes the Hopfield network as an example and shows that quantum parallelism can be obtained in this computation model as well. The author hopes that this will stimulate the thinking of readers involved in developing novel quantum devices.

2. The Hopfield Network as a Tool for Solving Combinatorial Optimization Problems

The Hopfield network is a computation model for solving combinatorial optimization problems that employs the operation of a specific recurrent network. (Hereafter we call the recurrent network itself a Hopfield network.) The concept of a Hopfield network is illustrated in Fig. 1. The network consists of threshold elements and connections. The connection weights W_{ij} and θ_i can be given any desired value, with the restrictions that $W_{ij} = W_{ji}$ and $W_{ii} = 0$. The outputs V_i of the threshold elements i wrap around to become the inputs to the network. Each threshold element i produces an output "1" if the weighted sum of inputs ($\Sigma W_{ij} X_i + \theta_i$) is positive and an output "0" if the weighted sum of inputs is negative. The point of this network is that, starting at a given initial position, it changes its *internal state* (a set of the outputs V_i of the threshold elements) to minimize the value of the *energy function* defined by

$$E = -1/2 \sum_{i \neq j} W_{ij} V_i V_j - \sum_i \theta_i V_i \,. \qquad (1)$$

By adjusting the connection weights we can relate the energy function of the network to the cost function of a given optimization problem. In this way, we can find the solution to the problem simply by observing the final state that the network reaches. For details, see Ref. 2.

Unfortunately, the correct solution cannot always be assured. This is because the basic Hopfield network has many states of locally minimum energy, in addition to the globally minimum state. In most cases the network will get stuck in a local minimum and a solution will not be reached. This is an unavoidable drawback in the Hopfield network and has limited its field of application. (This local-minima problem is a natural result of the fact that each occurrence of state transition in the threshold elements is independent of all others. It is unavoidable to the extent that we are tied to the classical concept of the Hopfield network.) To overcome this problem, we here consider designing a Hopfield network with single-electron circuits.

3. Single-Electron Circuit Structure for Implementing the Hopfield Network

The single-electron circuit changes its state to decrease its free energy. We can make use of this property to design a Hopfield network (see Ref. 3). The author proposes here a likely circuit design, as illustrated in Fig. 2. A tunnel junction with an excess electron is used as a threshold element (Fig. 2(a)); we define the state of the tunnel junction as "1" if the electron is on the right of the junction and as "0" if it is on the left. The connection between two tunnel junctions can be established by a pair of coupling capacitors (Fig. 2(b)); the connection weight can be set to either positive or negative, depending on the layout of the capacitor coupling. The overall configuration of the network is illustrated in Fig. 2(c). (An excess electron is also set on each bias node.) A ground capacitance exists between each node and ground (not illustrated here for simplicity). A sample set of capacitance

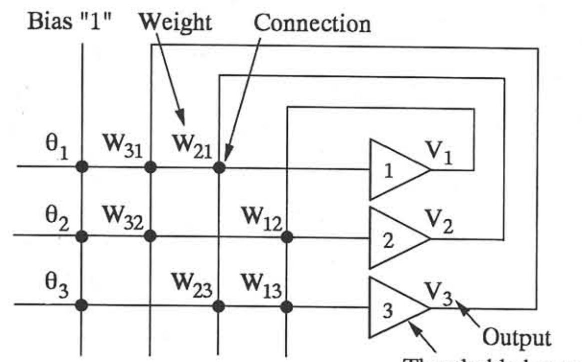

Fig. 1 Concept of the Hopfield network.

parameters is given in the figure.

Starting from a given initial position, the circuit changes its internal state (the arrangement of electrons) to minimize its free energy. The author has confirmed that the free energy for this circuit is given by

$$E = A - 1/2 \sum_{i \neq j} B_{ij} N_i N_j - \sum_{i} C_i N_i \ , \qquad (2)$$

where N_i is the state of each tunnel junction, and that the coefficients A, B_{ij}, and C_i can be set at any desired value, based on the connection pattern and the capacitance values of the tunnel junctions, connection capacitors, and ground capacitors. In this way we can be certain that the circuit will operate as a complete Hopfield network.

The internal state of this circuit is expressed by a set of the states of the tunnel junctions. For the sample circuit in Fig. 2(c), the internal state is expressed as (N_1, N_2, N_3). The energy values for all the possible internal states are compared in Fig. 3. The global minimum is state (0, 0, 0). Each solid arrow in the figure indicates the possible occurrence of a state transition due to a tunneling event (we here assume zero temperatures and therefore no energy excitation). States (1, 0, 1) and (0, 1, 1) have several incoming paths but no outgoing path; therefore these two states act as a local minimum. In this sense, these conditions do not differ from the classical Hopfield network.

4. Quantum Operation in the Hopfield Network, Using the Co-Tunneling Phenomenon

The local-minima problem comes from the fact that each of the electron tunnelings through the tunnel junctions occurs independently. To overcome this, we consider making good use of quantum phenomena. The point is that the novel Hopfield network can be attained if we can somehow build a single-electron Hopfield network such that *the co-tunneling phenomenon occurs frequently.* Co-tunneling is a phenomenon in which two or more tunneling events occur simultaneously in a form of coherent combination. Through the co-tunneling phenomenon, the sample circuit in Fig. 2(c), for example, can change its state from (1, 0, 1) and (0, 1, 1) to the global minimum (0, 0, 0), as illustrated by the dashed arrows; thus the local-minima problem disappears. Co-tunneling can be induced by increasing the tunnel junction conductance, so our concept is realizable.

We call this type of single-electron network a *quantum Hopfield network.* In this quantum Hopfield network, it is certain that, starting at a given initial state, the global minimum state can always be established. (Put another way, the network calculates simultaneously many energy values for all possible combinations of the junction states to find the minimum energy state. Therefore quantum parallelism is obtained, though in a form different from that of the quantum Turing machine.) Using this property, we can solve various combinatorial problems, including NP-complete problems, without being troubled by the local-minima problem.

An open question is whether it would be practical to build actual devices to perform such quantum network operation, or whether they would forever remain a thing of the imagination. Although various problems lie ahead, the author believes that theoretical and technological progress will sooner or later make such devices feasible. When the first quantum Hopfield network is built, we will then acquire a computing tool that can be used for solving NP-complete problems efficiently – a task that is difficult for every computation machine known today.

References

1) D. Deutsch: Proc. R. Soc. Lond. A400, (1985) 97.
2) J. J. Hopfield: Proc. Natl. Acad. Sci. USA 79, (1982) 2554
3) S. Bandyopadhyay and V. Roychowdhury, Jpn. J. Appl. Phys. 35, (1996) 3350.

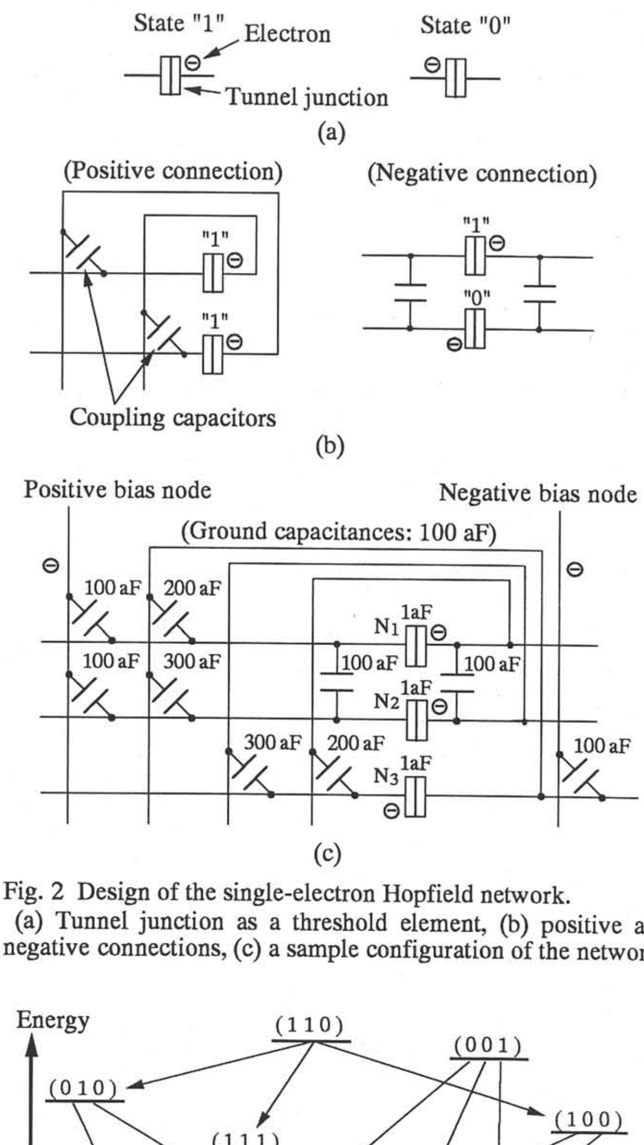

Fig. 2 Design of the single-electron Hopfield network.
(a) Tunnel junction as a threshold element, (b) positive and negative connections, (c) a sample configuration of the network.

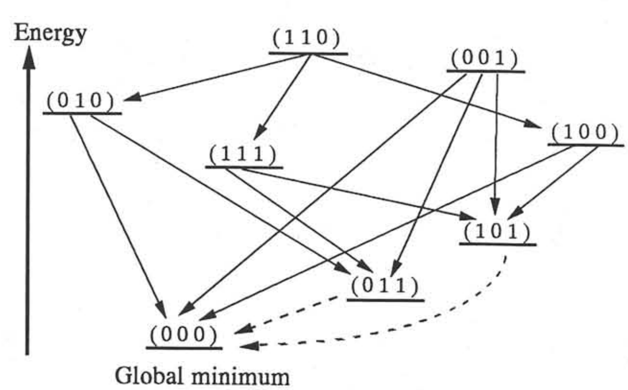

Fig. 3 Energy diagram for the single-electron Hopfield network in Fig. 2 (c).