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Analog Computation Using Quantum-Dot Spin Glass

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1. Introduction

One of the challenges in nanoelectronics is to develop information processing systems that utilize quantum mechanical effects. Analog computation using quantum structures is a promising architecture for information processing in nanoelectronics.[1)] This paper proposes an analog-computation system using the quantum-dot spin glass.

Analog computation is a processing method for solving a mathematical problem by applying an analogy of a physical system to the problem. To implement analog computation, we must prepare an appropriate physical system and use its physical quantity to represent each problem variable. If the mathematical relation between the physical quantities are analogous to those of the problem, then we can find the solution to the problem by observing the behavior of the system and measuring the physical quantities.

Spin glass is a kind of ferromagnetic-antiferromagnetic mixture.[2)] There is competition between the ferromagnetic and antiferromagnetic interactions in the spin glass. This property may make it possible to construct analog computation system that solves combinatorial optimization problems.[3)] In practice, however, analog computation that uses spin glass cannot be implemented because it is difficult to control the antiferromagnetic and ferromagnetic interactions in the spin glass.

In this paper we propose a novel analog-computation system using *quantum-dot spin glass*. We describe the spin-glass-like behavior in a two-dimensional (2D) array of quantum dots in Section 2, and apply an analogy of the quantum-dot spin glass to a combinatorial optimization problem in Section 3. We will demonstrate that the spin glass can perform analog computation.

2. Quantum-Dot Spin Glass

Technological advances have enabled us to fabricate an array of quantum dots. If an array is designed well, quantum-dot spin glass can be fabricated. Let us consider a 2D arrangement of quantum dots that are occupied by single electrons coupled to each other by strong correlation interaction. We can analyze the magnetic property of the quantum dot array given by the extended Hubbard model:

$$H = \sum_{k,j,\sigma} t_{kj} c^{+}_{k,\sigma} c_{j,\sigma} + \frac{1}{2} \sum_{k,j,\sigma} U_{kj} n_{k,\sigma} n_{j,-\sigma} + \frac{1}{2} \sum_{k,\sigma} U n_{k,\sigma} n_{k,-\sigma} \quad (1)$$

where $c_{k\sigma}^{+}$ ($c_{k\sigma}$) is the creation (annihilation) operator for an electron at the quantum dot k with spin σ; $n_{k,\sigma}$ is the number operator of the electron for the quantum dot k and spin σ; t_{kj} is the overlap integral that represents the interdot coupling between two quantum dots k and j; U_{kj} is Coulomb repulsion between the electrons at the k- and j-th dots; and U is the charging energy for a single quantum dot.

We first calculated the low-energy eigenstates of two electrons in two-dot and three-dot systems. Figure 1 shows the dependence of the singlet and triplet orbital levels on the amplitude of the overlap integral t. The diameter of the quantum dot is assumed to be 2 nm and the dielectric coefficient to be 10. The ground state of the two-dot system corresponds to the singlet (antiferromagnetic) state, in which each dot is occupied by one electron and the spins of the two electrons are antiparallel, as shown in Fig.1a. The exciting state is the triplet state in which the spins of the two electrons are parallel. Thus, we found that the exchange interaction coefficient J is negative and equal to half of the energy difference between the singlet and triplet states. The value of $|J|$ increases as the overlap integral increases. In the case of the three-dot system, owing to the Coulomb repulsion, the two electrons occupied the two dots in the left and right sides. The ground state of the three-dot system corresponds to the triplet (ferromagnetic) state, in which the spins of the two electrons are parallel (Fig.1b). The exciting state is the singlet state. J is positive. Though the central dot in the three-dot system is not occupied by an electron, it plays an important role in the ferromagnetic interaction.

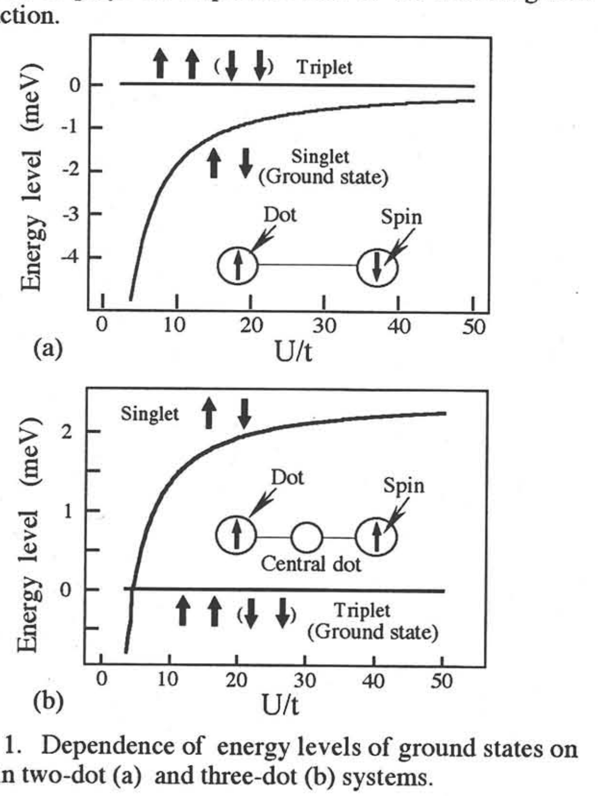

Fig. 1. Dependence of energy levels of ground states on t in two-dot (a) and three-dot (b) systems.

Next we designed a 2D array of quantum dots by mixing two-dot (antiferromagnetic interaction) and three-dot (ferromagnetic interaction) systems. The array consists of 20 quantum dots and 16 electrons. We analyzed the ground state of the quantum-dot array using the Monte-Carlo simulation method.[4] Figure 2 shows the simulated ground state of the quantum-dot array. There is competition between the ferromagnetic and antiferromagnetic interactions in the quantum-dot array. Because of the competition, no single configuration of the spins is uniquely favored by all the interactions. For example, the energy of the array does not change even if the quantum dot A takes the spin polarization represented by the arrow that is put in a parenthesis. This is called as "frustration". The result indicates that the quantum-dot array shows the spin glass behavior.

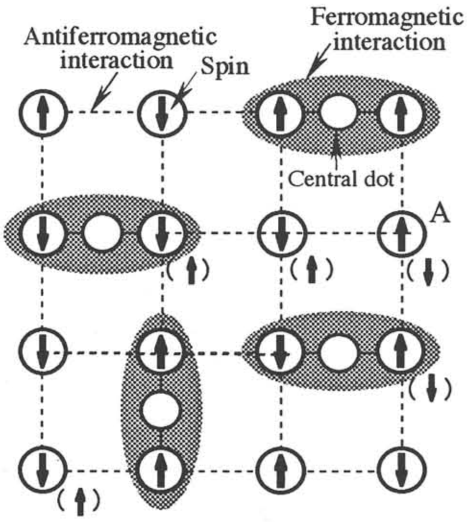

Fig. 2. Simulated ground state of quantum-dot array

3. Analog Computation

We tried mapping combinatorial optimization problems on the quantum-dot spin glass. Figure 3 shows an example of a max cut problem and the corresponding quantum-dot spin glass. Given a five-endpoint graph with positive weights on six branches, the max cut problem is defined as the problem of *finding a partition of the graph into two disjoint groups such that the sum of the weights of the branches that have two endpoints in two groups, respectively, is maximal.*

To solve this problem, we constructed a quantum-dot spin glass (Fig. 3b) by choosing appropriate exchange interaction coefficients J_i between the electrons (for the fabrication technique, see Ref. 5). We defined that the "up" polarization of the spin represents the endpoint in group 1 and "down" represents the endpoint in group 0. The energy function of the spin glass is analogous to the cost function of the max cut problem. Consequently, one of the optimal solutions can be obtained as long as the spin glass converges in its ground state.

We analyzed the ground state of the spin glass by the following Heisenberg model,

$$H = -2 \sum_{k \neq j} J_{kj} \mathbf{S}_k \cdot \mathbf{S}_j \ , \qquad (2)$$

where $\mathbf{S}_k$ and $\mathbf{S}_j$ are spin polarizations of the electrons at the k and j quantum dots, and J_{kj} is the exchange interaction coefficient between the spins of those electrons.

Figure 3c shows the typical development of the system energy for the quantum-dot spin glass. In the initial arrangement, the parallel electron spins in the quantum dot array at higher energy. Then the system energy decreases with changing eigenstate. Finally the spin system reaches one of its ground states. The ground state gives one of the optimal solutions to the max cut problem. The results show that the quantum-dot spin glass can perform analog computation.

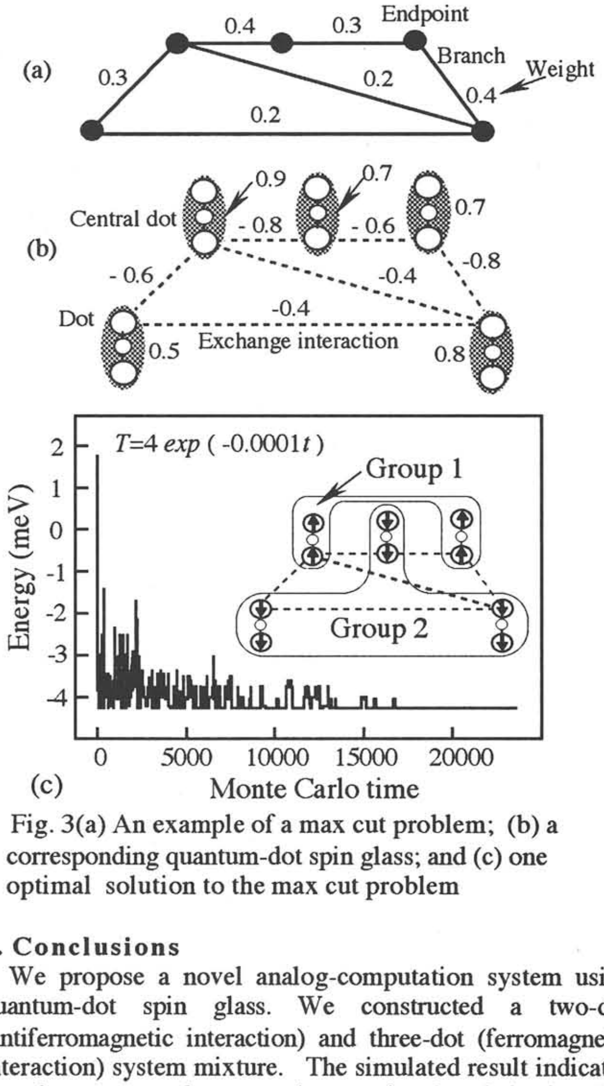

Fig. 3(a) An example of a max cut problem; (b) a corresponding quantum-dot spin glass; and (c) one optimal solution to the max cut problem

4. Conclusions

We propose a novel analog-computation system using quantum-dot spin glass. We constructed a two-dot (antiferromagnetic interaction) and three-dot (ferromagnetic interaction) system mixture. The simulated result indicates that the quantum-dot array shows spin glass behavior. We then mapped a max cut problem onto a quantum-dot spin glass and found its optimal solution. The results demonstrate that the quantum-dot spin glass can perform analog computation.

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