Analysis of a charged bio molecule particle passing through semi conductive Ion channel on the excitable biological membrane.

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1. Introduction.
Transportation of bio molecules across the biological membrane is essential for maintaining the physiological cellular function. Almost all the bio molecular particles carry electrical charges. As a result there is an ionic atmosphere around the bio molecular particle. During passing through the pore on the membrane, the motion of the bio molecular particles are influenced by the interaction between their charges and those, which are fixed on the inner surface of the channel pore. Thus the passing velocity and the potential around the bio molecular particle are strictly determined by the diffusion, fluid dynamics and electrical dynamics.

The present paper introduces a method for computing the analytical form of velocity and potential of charged bio molecular particle under the constant electrical field.

2. Assumptions.
We set following assumptions for mathematical treatment.
1. The electrophoresis velocity is proportional to the intensity of the applied field.
2. The electrolyte is composed of equal numbers of ions of equal but opposite charge. There is a surface phase on the solid side of the boundary in which the potential varies.
3. The thickness of the surface phase of the particle is small compared to the particle radius.
4. The charge in the surface phase is immobile and they cannot move laterally over the surface of the particle. There is no surface conductance effect.
5. The thickness of the double layer is sufficiently thin compared with the average distance or between neighboring particles.

We use coordinates fixed on the spherical particle so as to analyze the steady fluid motion. The equation of continuity is
\[
\text{div} (\mathbf{v}) = 0 \quad \text{-----(1)}
\]
The motion equation is expressed as
\[
\rho + \sigma \frac{\partial \sigma}{\partial t} = \mathbf{a} \mathbf{g} \mathbf{v} \quad \text{-----(2)}
\]
Where \( p \) is the pressure, \( \rho \) is the charge density and \( \sigma \) is the electrostatic potential, \( \mathbf{a} \) is the viscosity and \( \mathbf{v} \) is the fluid velocity. By setting mobility \( \sigma_i \) for the ionic velocity when a unit force is applied to it, the mean velocity \( u_i \) of type \( i \) ion is given by
\[
u_i = v - \sigma_i (z_i e \sigma + kT \frac{\partial m_i}{m_i}) \quad \text{-----(3)}
\]
Where \( v \) is fluid velocity, \( m_i \) is the concentration of the type \( i \) ion.

Equations (1) to (4) describe all the phenomena that are required to calculate the electrophoresis velocity of the particle. We refer to the indirect successive substituting method that has been introduced by Booth (1950).

3. Outline of the successive substitution method.
Step A. (3) and (4) are solved with \( v = 0 \) and \( E = 0 \). We denote the solution \( \sigma_1 \) for this condition by \( \sigma_1 \). It is a potential near the surface of the particle when the particle is at rest and no field is acting on.

Step B. We substitute \( \sigma_1 \) for \( \sigma \) in (2). This gives a solution \( \mathbf{v}_1 \) for the velocity. We can have the first estimate \( U_1 \) for the electrophoretic velocity \( U \) by adjusting the velocity at infinity so that the total force on the sphere vanishes.

Step C. We substitute \( \mathbf{v}_1 \) for \( \mathbf{v} \) in (3) and (4). Then the equations afford a higher order solution for \( \sigma_2 \) denoted as \( \sigma_2 \).

Step D. Repeating the steps B to C \( N \) times, we set \( \psi_1 \) equal to \( \sigma_2 \), we can have \( U_2 \).
Let a product we as the total charge on the particle. The order approximation of \( U \) can be expressed by

\[
U = c Q,
\]

where \( c \) is the coefficient with suffix denoting the order of approximation, and \( Q \) is the power of \( Q \). Since the physical value of \( Q \) can be available by experimental approach, particle velocity can be determined when the coefficient \( c \) has been determined.

4. Calculation of the first approximation to the potential.

In the first order approximation, there is no applied field. We apply the method introduced by Cronwall (1928) and developed by Booth (1950). The process starts from the Boltzmann distribution and Poisson Boltzmann equation of the system. On the basis of the equilibrium conditions, we have

\[
m_1 = n_i \exp(-e z_i/kT) \quad (6)
\]

where \( m_1 \) is the first approximation to the concentration of type \( i \) ion \( m_i \). \( n_i \) is the concentration of the type \( i \) ion at large distance from the particle. \( z_i \) satisfies the Poisson equation.

\[
\frac{\partial \rho_1}{\partial r} = -4 \pi e n_i z_i \exp(-e z_i/kT) \quad (7)
\]

The boundary conditions are given as

\[
\rho_1 \rightarrow 0, \quad r \rightarrow \infty \quad (8)
\]

and

\[
\frac{\partial \rho_1}{\partial r} = -e Q/(4 \pi a^2) \quad r \rightarrow a \quad (9)
\]

where \( r \) is the distance from the center of the sphere. In accordance with equation (5), we expand the potential \( \rho_1 \) around the sphere by the power series of charge \( Q \) with setting \( x = r/a \),

\[
\rho_1(x) = \sum_{n=0}^{\infty} a_n \left( \frac{x}{a} \right)^n \quad (10)
\]

The coefficient \( a_n \) are calculated to

\[
a_1(x) = \frac{b}{1+b} \exp(b-x) / x, \quad a_2(x) = 0, \quad a_3(x) = -q^3/6 \left( \frac{b}{1+b} \right)^3 \exp(3b-x) / x
\]

\[
\int e^{-u}/u \, du = \text{Ei}(u) \quad (11)
\]

5. Computed results.

The first order coefficient of velocity \( U_0 \) increased as the ratio between the radius of the particle against the double layer thickness has increased. The first order potential decreased rapidly as the distance from the center of the particle increased.

6. Discussion and Conclusion.

The present introduced only a method for calculating the passing velocity of a charged particle. A strong comparison will be required with biological measurement of the system.

References