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# Measurement of Two-Qubit States detected by Quantum Point Contacts

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## 1. Introduction

Quantum information processing in solid state nanostructures has attracted wide spread attention because of the potential scalability of such devices[1-3]. Recently, two-qubit coherent evolution and possibly entanglement have been observed in capacitively coupled Cooper pair boxes [4]. Since the realization of controlled two-qubit entanglement is regarded as a crucial milestone for the study of solid state quantum computing, it is important to search for the optimal configuration for a detector that is sensitive to two-qubit information, and to develop a proper formalism to study two-qubit measurement processes. In this paper we study a particular scheme for the quantum measurement of two charge qubits by quantum point contacts (QPCs) that are capacitively coupled to the qubits composed of quantum dots (QDs)(Fig.1).

## 2. Formulation

The Hamiltonian for the combined two qubits and the QPCs is written as $H = H_{\rm qb}{+}H_{\rm qpc}{+}H_{\rm int}$. $H_{\rm qb}$ is the qubit part which can be written as $H_{\rm qb} = \sum_{\alpha=L,R}(\Omega_\alpha\sigma_{\alpha x} + \Delta_\alpha\sigma_{\alpha z}) + J\sigma_{Lz}\sigma_{Rz}$,where $\Omega_\alpha$ and $\Delta_\alpha$ are the inter-QD tunnel coupling and energy difference (gate bias) in the qubit. Here we use the spin notation such that $\sigma_{\alpha x} \equiv a_\alpha^\dagger b_\alpha + b_\alpha^\dagger a_\alpha$ and $\sigma_{\alpha z} \equiv a_\alpha^\dagger a_\alpha - b_\alpha^\dagger b_\alpha$ ($\alpha = L, R$), where $a_\alpha$ and $b_\alpha$ are the annihilation operators of an electron in the upper and lower QDs of each qubit. $J$ is a coupling constant between the two qubits, originating from capacitive couplings in the QD system [5]. $H_{\rm qpc}$ is the QPC part:

$$H_{\rm qpc}{=}\sum_{\alpha=L,R}\sum_{i_\alpha s=\uparrow,\downarrow}\left(E_{i_\alpha}c_{i_\alpha s}^\dagger c_{i_\alpha s}{+}V_{\alpha s}(t)(c_{i_\alpha s}^\dagger d_s + d_s^\dagger c_{i_\alpha s})\right) + \sum_{s=\uparrow,\downarrow}E_d d_s^\dagger d_s + U d_\uparrow^\dagger d_\uparrow d_\downarrow^\dagger d_\downarrow\,. \quad (1)$$

Here $c_{i_L s}(c_{i_R s})$ is the annihilation operator of an electron in the $i_L$th ($i_R$th) level ($i_L(i_R) = 1, ..., n$) of the left(right) electrode, and $d_s$ is the electron annihilation operator of the QPC island. $H_{\rm int}$ is the interaction between the qubits and the QPCs.

$$H_{\rm int} = \sum_{\alpha=L,R}\sum_{i_\alpha,s}\delta V_{\alpha s}(t)(c_{\alpha s}^\dagger d_s + d_s^\dagger c_{i_\alpha s})\sigma_{\alpha z} \qquad (2)$$

Through this interaction, qubit states $|\downarrow\downarrow\rangle \equiv |A\rangle$, $|\downarrow\uparrow\rangle \equiv |B\rangle$, $|\uparrow\downarrow\rangle \equiv |C\rangle$, and $|\uparrow\uparrow\rangle \equiv |D\rangle$ can vary the QPC tunneling rate in the form of $\Gamma_\alpha^{(\pm)} \equiv 2\pi\rho_\alpha(E)|V_\alpha^{(\pm)}(E)|^2$ and $\Gamma_\alpha^{(\pm)'} \equiv 2\pi\rho_\alpha(E{+}U)|V_\alpha^{(\pm)}(E{+}U)|^2$, where $V_\alpha^{(\pm)} = V_\alpha{\pm}\delta V_\alpha$, and $\rho_\alpha(E)$ is the density of states of the electrodes ($\alpha = L, R$). We construct the equations of qubits-QPCs density matrix elements at $T = 0$, following the procedure developed by Gurvitz[6]. The wave function $|\Psi(t)\rangle$ of the system can be expanded over the states of the qubits and the island between the two QPCs (Fig.2). This method is applicable as long as the energy-levels of the island is inside the chemical potentials of both electrodes. Assuming that there is no magnetic field, after lengthy calculations, we obtain 48 equations for the density matrix elements $\rho_{z_1 z_2}^u(t)$ ($u$ indicate quantum states of the detector (Fig.2) and $z_1, z_2 = A, B, C, D$ are those of the qubits) as

$$\begin{aligned}\frac{d\rho_{AA}^a}{dt} &= -2\Gamma_L^{(-)}\rho_{AA}^a - i\Omega_R(\rho_{BA}^a - \rho_{AB}^a) - i\Omega_L(\rho_{CA}^a - \rho_{AC}^a) \\ &\quad + \Gamma_R^{(-)}(\rho_{AA}^{b\uparrow} + \rho_{AA}^{b\downarrow}) \\ \frac{d\rho_{AB}^a}{dt} &= (i[J_B - J_A] - 2\Gamma_L^{(-)})\rho_{AB}^a - i\Omega_R(\rho_{BB}^a - \rho_{AA}^a) \\ &\quad - i\Omega_L(\rho_{CB}^a - \rho_{AD}^a) + \sqrt{\Gamma_R^{(-)}\Gamma_R^{(+)}}(\rho_{AB}^{b\uparrow} + \rho_{AB}^{b\downarrow}) \\ &\quad .....\end{aligned} \qquad (3)$$

where $J_A{=}\Delta_L{+}\Delta_R{+}J$, $J_B{=}\Delta_L{-}\Delta_R{-}J$, $J_C{=}{-}\Delta_L{+}\Delta_R{-}J$, $J_D{=}{-}\Delta_L{-}\Delta_R{+}J$. The measurement strength on the qubits can be estimated by the dephasing rate as $\Gamma_d^{(\alpha)} \equiv (\sqrt{\Gamma_\alpha^{(+)}} - \sqrt{\Gamma_\alpha^{(-)}})^2$. The readout current is obtained as[6]

$$I(t) = e\Gamma_R[\rho^{b\uparrow}(t) + \rho^{b\downarrow}(t)] + 2e\Gamma'_R\rho^c(t). \qquad (4)$$

where $\rho^u$ is given by $\rho^u \equiv \rho_{AA}^u{+}\rho_{BB}^u{+}\rho_{CC}^u{+}\rho_{DD}^u$. We choose $\Gamma_A^L = \Gamma_B^L = \Gamma_A^R = \Gamma_C^R = \Gamma^{(-)} = 0.8\Gamma$, $\Gamma_C^L = \Gamma_D^L = \Gamma_B^R = \Gamma_D^R = \Gamma^{(+)} = 1.2\Gamma$, which lead to $\Gamma_d \sim 0.04\Gamma$ as a typical case ($\Gamma$ is a tunneling rate without qubits).

## 3. Numerical results

Figure 3 shows the time-dependent current near $t \sim 0$ assuming that the two qubits are initially in either of the four product states. At small $t$ state $|A\rangle$ suppresses the current the most while state $|D\rangle$ the least. Thus we can distinguish the four product states by the readout current. Hereafter we will focus on the $J = 0$ case.

In quantum Zeno effect, a continuous measurement slows down transitions between quantum states due to the collapse of the wave function into observed states. We show the Zeno effect of two qubits in Fig 4, where the initial state is $|D\rangle$ state ($\rho_{DD}(t = 0) = 1$). As the measurement strength increases ($\Gamma_d$ increases), the oscillations of density matrices of qubits are delayed, which is a clear evidence of the slowdown described by the Zeno effect.

Strong measurement destroys entangled states such as the Bell states: $|e_1\rangle{=}(|\downarrow\downarrow\rangle{+}|\uparrow\uparrow\rangle)/\sqrt{2}$, $|e_2\rangle{=}(|\downarrow\downarrow\rangle{-}|\uparrow\uparrow\rangle)/\sqrt{2}$, $|e_3\rangle{=}(|\downarrow\uparrow\rangle{+}|\uparrow\downarrow\rangle)/\sqrt{2}$, and $|e_4\rangle{=}(|\downarrow\uparrow\rangle{-}|\uparrow\downarrow\rangle)/\sqrt{2}$. The measure of entanglement is estimated from the reduced density matrix of the qubits using the concept of concurrence[7].Figure 5 shows the relation between the time-dependent concurrence of the singlet state ($|e_4\rangle(= (|B\rangle - |C\rangle)/\sqrt{2})$) and $\Gamma_d$, showing that the strong measurement degrades the entanglement quickly.

In charge qubits, the wave functions of the entangled states is expected to extend over the qubits compared with the product states. For example, the density matrix equations for a singlet state $|e_4\rangle$ of free qubits($H_{\rm int} = 0$) satisfy $\dot\rho_{BB} + \dot\rho_{CC} - \dot\rho_{BC} - \dot\rho_{CB} = 0$ ($\Gamma_\alpha^{(+)} = \Gamma_\alpha^{(-)}$ in Eq.3). This suggests that the charge distribution of the singlet state is less effective on influencing the readout current. Indeed we found that the readout currents of the entangled states are uniform compared with the product states shown in Fig.6. These features hold as long as the entangled states are close to the Bell states. Figure 7 shows the time-dependent current of the generalized singlet state $\cos\theta|\uparrow\downarrow\rangle + e^{i\varphi}\sin\theta|\uparrow\downarrow\rangle$ in the range of $\varphi = \pi$, $0 \leqq \theta \leqq \pi/2$.

## 4. Conclusion

We have solved master equations and described various time-dependent measurement processes of two charge qubits by two QPCs. The current through the QPCs is shown to be an effective means to measure results of quantum calculations and entangled states.

## References


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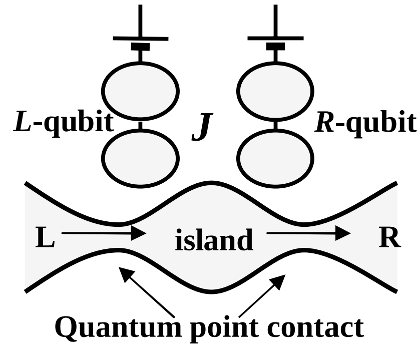


**Fig.1: Qubits are capacitively coupled to QPCs as a detector.**

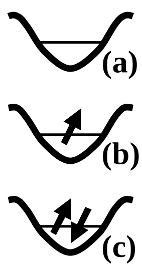


**Fig.2: Electronic states of an island between the QPCs.**

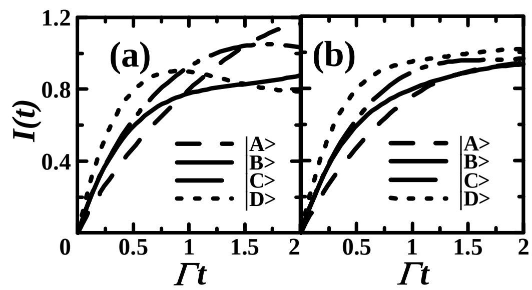


**Fig.3: Time dependent current of four product qubit states. $|A\rangle = |\downarrow\downarrow\rangle$, $|B\rangle = |\downarrow\uparrow\rangle$, $|C\rangle = |\uparrow\downarrow\rangle$, $|D\rangle = |\uparrow\uparrow\rangle$. $\Omega_L = \Omega_R = 0.75\Gamma$, $\Gamma_d = 0.04\Gamma$. (a) J=0, (b) J=Γ.**

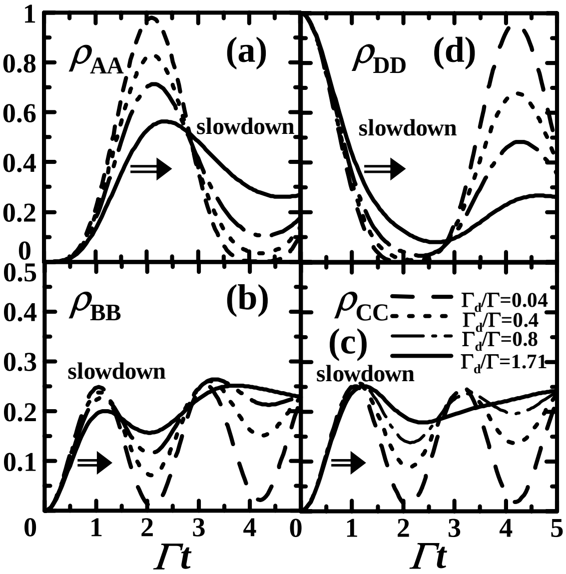


**Fig.4: Time dependence of $\rho_{AA}$, $\rho_{BB}$, $\rho_{CC}$ and $\rho_{DD}$ when the strength of measurement $\Gamma_d$ increases. $\Omega_L = \Omega_R = 0.75\Gamma$, J=0. As $\Gamma_d$ increases, the coherent motions of qubits slow down (Zeno effect).**

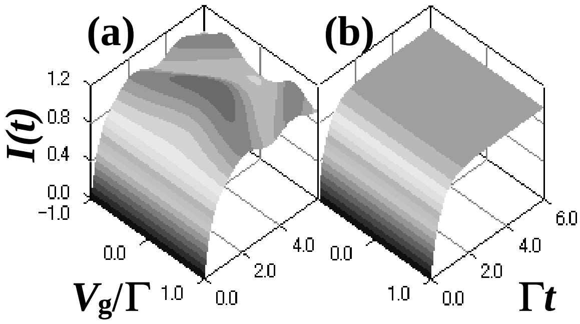


**Fig.6: Time dependent current of $|B\rangle$ state ((a)) and singlet state $|e_4\rangle$ ((b)) when $Vg(=\Delta_L=\Delta_R)$ changes. $\Omega_L = \Omega_R = 0.75\Gamma$, J=0, $\Gamma_d = 0.04\Gamma$. The singlet state is robust to the measurement.**

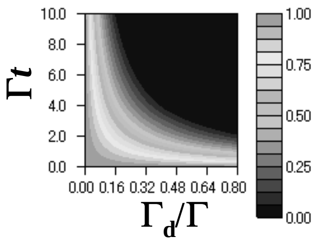


**Fig.5: Time dependent concurrence of a singlet state $|e_4\rangle$ as a function of the dephasing rate $\Gamma_d$.**

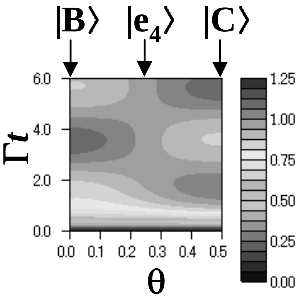


**Fig.7: Time dependent current from $|B\rangle$ state to $|C\rangle$ state through singlet state ($|e_4\rangle$). $\Omega_L = \Omega_R = 0.75\Gamma$, J=0, $\Gamma_d = 0.04\Gamma$.**