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Ohm's Law from a Transmission Viewpoint

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1. Introduction

Ohm's law is an established basic law. In the theoretical derivation of the law, uniformity of the system slightly shifted from equilibrium, as well as the relaxation time approximation, is usually assumed. In a nanoscale conductor placed between biased electrodes with a constant electric field in between, however, the carrier has the potential energy that increases toward the higher potential electrode, and the fact may cause a break of the uniformity hypothesis.

In this paper, we have analyzed transport property of a nanoscale conductor placed between the source and the drain electrode by use of exact solution of the one-dimensional Boltzmann transport equation (BTE). Some modifications in elastic transport theory have become evident in nanoscale geometry.

2. Analysis of an elastic conductor

In the conventional framework where the relaxation time approximation is assumed, the mean carrier velocity is analyzed by the Newton equation in Fig. 1, and the mobility μ is extracted. Multiplying the carrier density *n* as well as the carrier charge, yields the conductance, and the Ohm's law is derived. Uniformity hypothesis assures all these quantities are uniform in the conductor. The local current value is described also by local parameters. In microscopic description of the transport between biased electrodes, however, analysis based on the transmission viewpoint (Fig.2), where destination of each carrier is pursued as in BTE approach, is necessary.

We consider elastic scatterings first. In a one-dimensional conductor extended along *x*-axis, the allowed scattering is only between *k* and –*k* at the same *x* point. The transport in an electric field *E* is simply described by the BTE shown in Fig. 3, where τ is the elastic scattering time. Solution of the equation is derived as $f(x,k)$ and $f(x,-k)$ in Fig.3, where *C* and *D* are constants of integration and ε is the incident carrier energy. With use of the solution, we can derive the transmission coefficient *T* from source to drain through the device length *L* as

$$T = \frac{qE\tau}{qE\tau + \sqrt{2m}\left(\sqrt{qEL+\varepsilon} - \sqrt{\varepsilon}\right)}.$$

We can also compute the carrier density *n*(*x*), carrier mobility μ=<*v*(*x*)>/*E*, where <*v*(*x*)> is the mean velocity at *x*, and the device current.

Fig. 4 shows distribution of the carrier density *n*(*x*) normalized by the injected carrier density with energy *kT* for various *L* values. At the source edge, it is 2 due to presence of back-scattered flow. Fig. 5 shows distribution of mobility μ=<*v*(*x*)>/*E* as well as the conventional value $q\tau/2m^*$. *n*(*x*) decreases while the mobility (velocity) increases toward the drain. Carrier number decreases toward drain due to perpetual backscatter action toward source. The velocity increases toward drain so as to achieve the current continuity. Anyway, the uniformity of these quantities assumed by the conventional theory no longer applies. Fig. 6 shows the transmission coefficient from source to drain as a function of electric field. The transmission decreases as *L* is increased, and eventually vanishes for $L\to\infty$. The current density for one-dimensional non-degenerate semiconductor at room temperature is evaluated, and is plotted in Fig. 7 as a function of the electric field. The current is proportional to *E* for small electric field values as is in Ohm' law, but the range depends on the size *L*. We can verify that the Ohm's law range is bounded by the bias voltage $EL \leq kT/q$, an extremely small value. Beyond the range, the current density is roughly proportional to $E^{1/2}$, and approaches the injected current flux for $E\to\infty$. On the other hand, the current density approaches zero for $L\to\infty$. These properties are readily derived from the expression of transmission *T*. It is surprising that the Ohmic region is so narrow, and that the current density is not only controlled by the local field but also by the global size of conductor *L*.

3. Consideration of inelastic scatterings

Real conductors include inelastic scatterings. We can employ the modeling of inelastic scattering as follows. The acoustic phonon scattering is included in the elastic scattering as usually assumed. The energy relaxation is mainly born by the optical phonon emission with dissipation of finite energy $\hbar\omega_0$, which is a few times of *kT*. The rare absorption is negligible. Carriers in Boltzmann distribution are injected from source to channel with the typical energy *kT*. Before their kinetic energy increases over $\hbar\omega_0$ by E there occurs no inelastic scattering. Beyond, the optical phonon emission occurs and reduces the carrier energy by

$\hbar\omega_0$. Fig. 8 shows the current density obtained by the analysis including the inelastic scattering. Surprisingly, the Ohm's law with the current proportional to *E* is restored for a wide region, and the current saturation at sufficiently large electric field appears. Fig. 9 is the famous experimental report by Ryder and Shockley a half century ago, showing the typical velocity saturation, but actually showing the current saturation. Close resemblance of our result in Fig. 8 to the experiment, in spite of the discrepancy in elastic theory, suggests us to re-examine the mechanism of Ohmic current and current saturation.

4. Conclusion

Carrier transport in a constant electric field is analyzed by solving the one-dimensional BTE. In the case where only the elastic scattering is considered and no energy relaxation is assumed, discrepancy from the conventional mobility theory is remarkable in some points. When the inelastic scattering is taken into account, however, electric characteristics consistent with the conventional theory are restored.

Reference

[1] E. J. Ryder, Phys.Rev., **99** (1953)766.

Newton's equation — Steady state

$$m\frac{d\mathbf{v}}{dt}=q\mathbf{E}-m\frac{\mathbf{v}}{\tau} \Rightarrow \mathbf{v}=\frac{q\tau}{m}\mathbf{E}$$

$$\mu=\frac{q\tau}{m},\quad \mathbf{v}=\mu\mathbf{E},\quad \mathbf{i}=qn\mu\mathbf{E}$$

Fig. 1. Conventional mobility theory is derived from Newton equation.

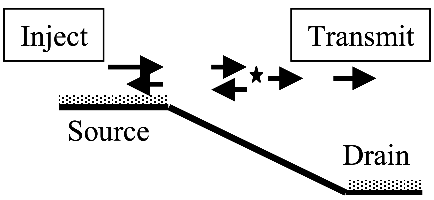

Fig. 2 A transmission viewpont pursuing each particle's destination.

$$\frac{q}{\hbar}E\frac{\partial f(x,k)}{\partial k}+\frac{\hbar k}{m}\frac{\partial f(x,k)}{\partial x}+\frac{1}{\tau}\left(f(x,k)-f(x,-k)\right)=0$$

$$f(x,k)=D-\frac{C}{\tau}\sqrt{\frac{2m}{qE}\left(x+\frac{\varepsilon}{qE}\right)}\,\delta\left(\frac{\hbar^2k^2}{2m}-qEx-\varepsilon\right),$$

$$f(x,-k)=D-C\left(1+\frac{1}{\tau}\sqrt{\frac{2m}{qE}\left(x+\frac{\varepsilon}{qE}\right)}\right)\delta\left(\frac{\hbar^2k^2}{2m}-qEx-\varepsilon\right)$$

Fig. 3. One-dimensional BTE for elastic conductor, and its solution.

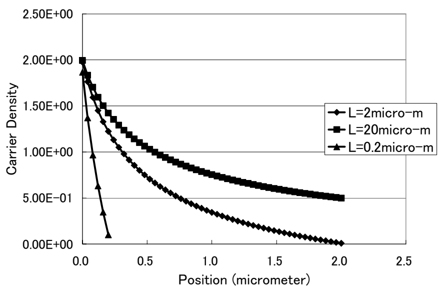

Fig. 4 Carrier density distribution for a elastic conductor.

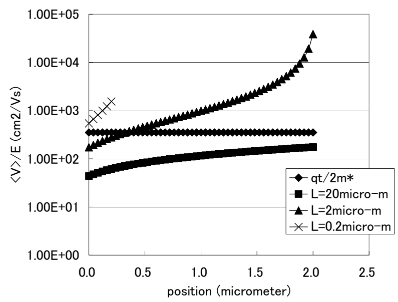

Fig. 5 "mobility" distribution for an elastic conductor.

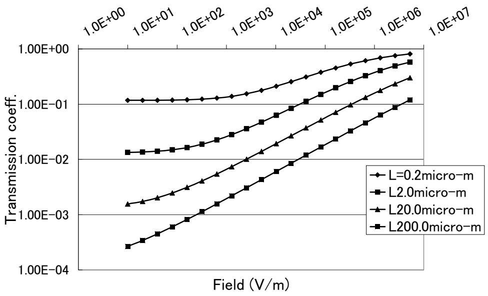

Fig. 6 Transmission coefficient as a function of field for an elastic conductor.

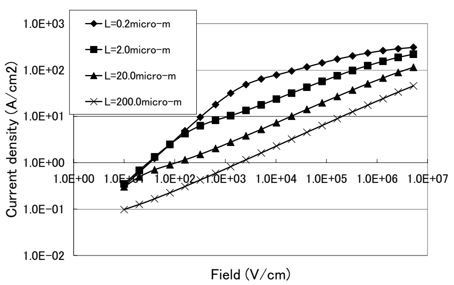

Fig. 7 Device current of an elastic conductor. L is the source drain distance.

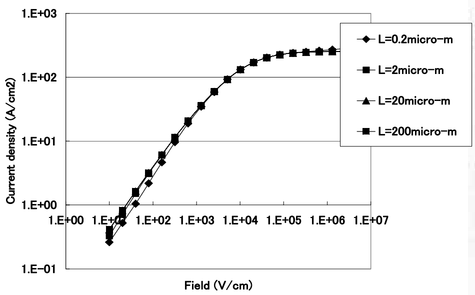

Fig. 8 Device current for the case the optical phonon emission is included.

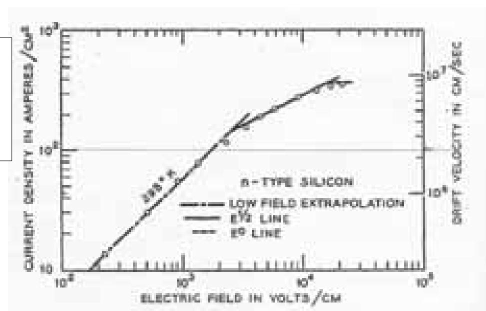

Fig.9 Ryder's experimental result showing device current for silicon in high field.