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Analytic Parameter Extraction of On-chip Spiral Inductors Using a Modified Skin Effect Model

Sanghoon Lee, Heesauk Jhon, Ickhyun Song and Hyungcheol Shin
Nano Systems Institute, Inter-University Semiconductor Research Center (ISRC) and School of Electrical Engineering, Seoul National University, Sillim-dong, Gwanak-gu, Seoul, 151-742, Republic of Korea.
Phone: +82-2-880-7282, e-mail: ppolook@naver.com

1. Introduction

The great parts of reported inductor models are non-physical and their parameter extraction is time-consuming due to curve fitting and parameter optimization by circuit simulators [1]. Therefore, some simple on-chip inductor models have been proposed in the previous research through the analytic methods [2, 3]. However, complex quadratic equations should be solved to obtain the elements related the skin effect and hence the extracted parameters are somewhat inexact. In this work, we present a simple and robust technique to extract each element in a modified skin effect model. The proposed model is confirmed by measuring inductors having different geometric parameters up to 40 GHz.

2. Equivalent Circuit Model

Fig. 2 illustrates the equivalent circuits of the proposed model. This model consists of series components (L_{s0}, L_{s1}, R_{s0}, R_{s1}) representing inductance and resistance of a signal line, oxide capacitance (C_{ox}) between top metal and silicon substrate, two shunt branches (R_{si}, C_{si}), which are substrate parasitic components, and substrate capacitance (C_{sub}) from the lateral substrate coupling at high frequency. In comparison with the previous work [2][4], the series combination of R_{s1} and L_{s1} are altered into the parallel ones. Therefore, increment of resistance due to both skin and proximity effects at high frequency can be simply and exactly obtained from the simple linear regression without losing the physical meaning. Using the above configuration, the series inductance, L_{s0}, which was underestimated due to excessive approximation in the previous work, is also accurately extracted.

3. Parameter Extraction Procedure

Series components of the model can be expressed by real and imaginary parts of $-1/Y_{21}$. At low frequency, where it is weakly affected by skin and proximity effects, impedance of L_{s1} can be considered as zero. That is, the real part of the series components can be expressed by only R_{s0}. Therefore, R_{s0} is directly extracted at low frequency region of the real part of $-1/Y_{12}$ as shown in Fig. 3. In order to extract R_{s1} and L_{s1}, real($-1/Y_{21}$) should be rearranged as follows.

$$\mathrm{Re}\left[\frac{-1}{Y_{21}}\right] = R_{s0} + \frac{\varpi^2 L_{s1}^2 R_{s1}}{R_{s1}^2 + \varpi^2 L_{s1}^2} \tag{1}$$

$$\frac{1}{\mathrm{Re}\left[\frac{-1}{Y_{21}}\right] - R_{s0}} = \frac{R_{s1}}{L_{s1}^2}\varpi^{-2} + \frac{1}{R_{s1}} \tag{2}$$

R_{s1} can be obtained from the inverse Y-intercept as shown in Fig. 4. L_{s1} is obtained from the extracted R_{s1} and linear slope. L_{s0} can be determined from the low frequency data of imaginary($-1/Y_{21}$) as follows.

$$\frac{\mathrm{Im}\left[\frac{-1}{Y_{21}}\right]}{\varpi} = \frac{L_{s1}R_{s1}^2}{R_{s1}^2 + \varpi^2 L_{s1}^2} + L_{s0} \cong L_{s1} + L_{s0} \tag{3}$$

The extracted value of $L_{s1}+L_{s0}$ is shown in Fig. 5.

C_{ox} is extracted with the conventional π-model. The impedances of C_{si} and C_{sub} are negligible at low frequency. Hence, C_{ox1} and C_{ox2} are obtained from the low frequency data of $Y_{11}+Y_{12}$ and $Y_{22}+Y_{12}$, using the followings.

$$C_{ox1} = -\frac{1}{\varpi\,\mathrm{Im}\left[\frac{1}{Y_{11}+Y_{12}}\right]} \tag{4}$$

Substrate parasitic elements are extracted using the following equation [3].

$$\frac{1}{real(Y_{sub})}\omega^2 = \frac{1}{R_{si}C_{ox1}^2} + \frac{R_{si}(C_{ox}+C_{sub})}{C_{ox1}^2}\varpi^2 \tag{5}$$

Here, Y_{sub}, which represents $Y_{11}+Y_{12}$, is the admittance of the shunt branches containing C_{ox}, C_{si} and R_{si}. R_{si} and C_{si} can be obtained from the slope and intercept of ω^2/real(Y_{sub}) as a function of ω^2 (Fig. 6).

C_{sub} can be extracted using the resonance frequency of parallel LC as follows.

$$C_{sub} = \frac{C_{ox1}C_{ox2}}{4\pi^2(L_{s0})(\mathrm{RF})^2 C_{ox1}C_{ox2} - C_{ox1} - C_{ox2}} \tag{6}$$

$$\text{Resonance frequency (RF)} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}} \tag{7}$$

The extracted model parameters are listed in Table I.

4. Model Verification

The proposed model is verified by comparing it with the measurement data in terms of Q-Factor, effective inductance, effective series resistance and S-parameters over a wideband up to 40 GHz.

Fig. 7 indicates the good matching between the proposed model and the measured Q-factor and demonstrates the applicability of the compact inductor model at millimeter wave. Fig. 8-9 represent the broadband accuracy of the effective inductance and effective series resistance for all inductors having various geometric parameters. It is also confirmed that the modified skin effect model defines the increment of the series resistance well by the skin and proximity effects. The root mean square (RMS) deviations of S-parameters with the different frequency ranges are listed in Table II.

5. Conclusion

The newly proposed analytic parameter extraction method for spiral inductors at mm-wave has been verified. The broadband accuracy has been confirmed with Q-factor, effective inductance and ESR, as well as the measured S-parameters up to 40 GHz. Therefore, the modified model can facilitate to extract various parameters of an inductor.

Acknowledgements

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References

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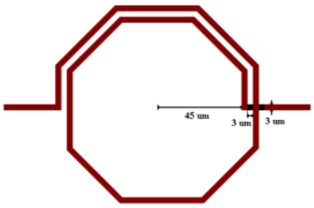

Fig. 1. Geometric parameters of the reference DUT.

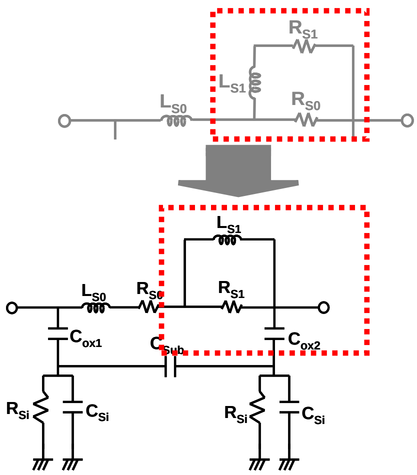

Fig. 2. The proposed equivalent circuit with the modified skin effect model.

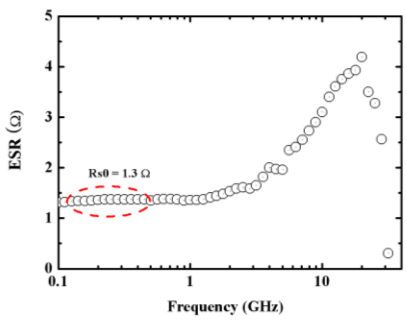

Fig. 3. R_{s0} is extracted at low frequency region of the measured series resistance (real($-1/Y_{12}$)).

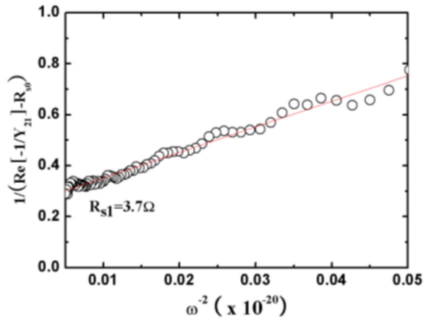

Fig. 4. Plot of $1/(\mathrm{Re}[-1/Y_{21}]-R_{s0})$ as a function of inverse ω square. R_{s0} and L_{s1} can be calculated from the Y-intercept and the linear slop.

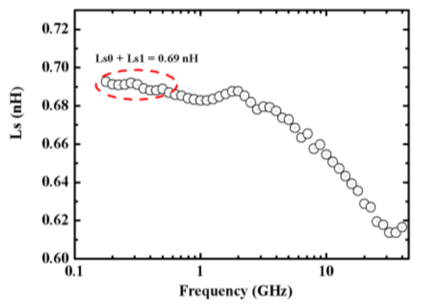

Fig. 5. Plot of image($-1/Y_{12}$)/ω as a function of frequency. In low frequency region, image($-1/Y_{12}$)/ω represents $L_{s0}+L_{s1}$ value.

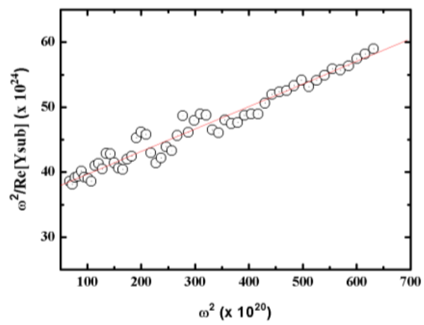

Fig 6. ω^2/real(Y_{sub}) as a function of ω square. R_{si} and C_{si} can be extracted from the Y-intercept and linear slop. Measured Y_{sub} is $Y_{11}+Y_{12}$.

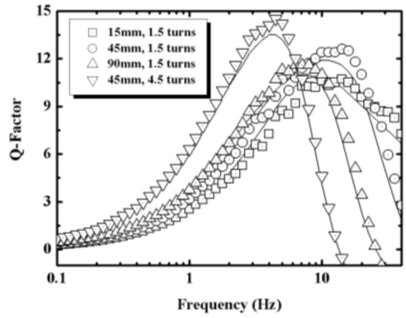

Fig. 7. Measured and modeled quality factors Q = -imag(Y_{11}) / real(Y_{11})

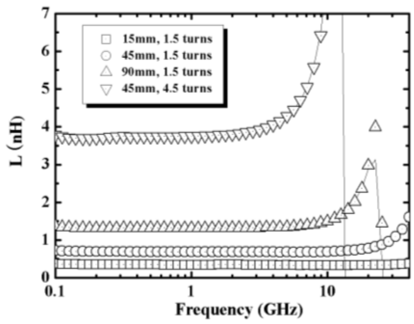

Fig. 8. Measured and modeled series inductance up to 40GHz

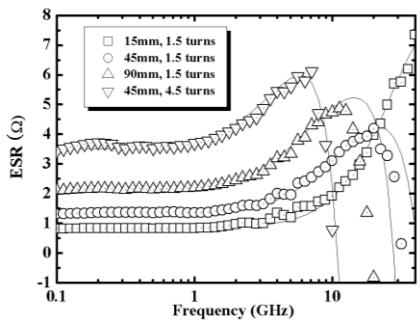

Fig. 9. Measured and model effective series resistance

Table. I. Extracted model parameters

DUT Num. (Inner Radius / # of turn)	Rs0 (Ω)	Rs1 (Ω)	Ls0 (pH)	Ls1 (pH)	Cox1 (fF)	Cox2 (fF)	Rsi (Ω)	Csi (fF)	Csub (fF)
1 (15 / 1.5)	0.83	10	281	54	10	10	111	18	5
2 (45 / 1.5)	1.3	3.7	630	63	20	20	70	24	11
3 (90 / 1.5)	2.1	4	1210	98	36	36	45	28	14
4 (45 / 4.5)	3.5	4	3560	166	43	43	60	29	83

Table. II. The root mean square (RMS) errors of the test structures for S-parameters

DUT Number (Inner Radius / # of turn)	F_{SR} (Self Resonant Frequency, GHz)	RMS error (%)		
		20 GHz	30 GHz	40 GHz
1 (15 / 1.5)	95	3.84	4.09	4.38
2 (45 / 1.5)	45	3.14	3.40	4.02
3 (90 / 1.5)	22	4.30	4.03	5.08
4 (45 / 4.5)	12	1.00	3.57	8.10