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Robustness of charge-qubit cluster states to double quantum point contact measurement

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I. INTRODUCTION

One-way quantum computing [1] is an important approach for quantum computation based on a series of one-qubit measurements starting from a cluster state of a qubit array. Cluster states are highly-entangled states involving all qubits and are typically generated from an Ising-like Hamiltonian, starting from an initial product state $|\Psi_0\rangle \equiv |\Psi(t=0)\rangle = \Pi_{i=1}^{N}|+\rangle_i$, where $|\pm\rangle_i = (|0\rangle_i \pm |1\rangle_i)/\sqrt{2}$. Here, $|0\rangle_i$ and $|1\rangle_i$ are the two states of the i-th qubit in an N-qubit system. In Ref. [2], we showed that cluster states in charge qubits can be created by applying a single gate bias pulse, right after preparing the initial product state (one-step generation method), and are more robust against nonuniformities among qubits than decoherence-free (DF) states [3] under a noise environment generated by a quantum point contact (QPC) detector. Measurements induce decoherence. Here, we investigate robustness of cluster states in charge qubits measured by double QPC (DQPC) detector with an island (discrete energy state) (Fig. 1). The island between DQPC models a trap site which is often unavoidable in solid-state qubits due to their small fabrication size. Thus, this setup constitutes a harsher and more realistic condition for qubits than that of Ref .[2], and we can test the robustness of cluster states by charge qubits. Here, we calculate a time-dependent fidelity of four charge qubits based on quantum dots (QD) with DQPC by solving density matrix (DM) equations.

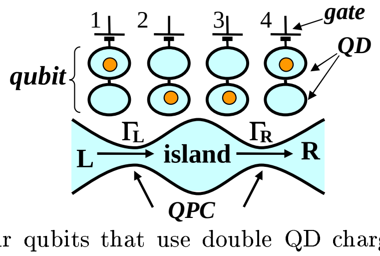

FIG. 1: Four qubits that use double QD charge states are capacitively coupled to a double QPC detector with an island. One excess charge is injected into each qubit. Single energy level is assumed in each QD and the island.

II. DENSITY MATRIX EQUATIONS

Hamiltonian for an array of charge qubits with nearest-neighbor interactions is expressed by [2] $H_{\rm cq} = \sum_i(\Omega_i\,\sigma_{ix} + \epsilon_i\,\sigma_{iz}) + \sum_{i<j} J_{ij}\,\sigma_{iz}\,\sigma_{jz}$, where σ_{ix} and σ_{iz} are Pauli matrices for the i-th qubit. Ω_i is the inter-QD tunnel coupling for coupled QD systems [4–6]. ϵ_i is the charging energy and corresponds to the energy difference between $|0\rangle_i$ and $|1\rangle_i$ for each qubit. The coupling constants J_{ij} are derived from the capacitance couplings [6].

The DM equations of four qubits and the DQPC detector at zero temperature of Fig. 1 are derived similarly to Ref. [7] by

$$\frac{d\rho^a_{z_1z_2}}{dt} = (i[J_{z_2}-J_{z_1}] - [\Gamma_L^{z_1}+\Gamma_L^{z_2}])\rho^a_{z_1z_2}$$
$$- i\sum_{j=1}^{N}\Omega_j(\rho^a_{g_j(z_1),z_2} - \rho^a_{z_1,g_j(z_2)}) + \sqrt{\Gamma_R^{z_1}\Gamma_R^{z_2}}(\rho^{b\uparrow}_{z_1z_2} + \rho^{b\downarrow}_{z_1z_2}),$$
$$\frac{d\rho^{b_s}_{z_1z_2}}{dt} = \left(i[J_{z_2}-J_{z_1}] - \frac{\Gamma_L^{z_1'}+\Gamma_L^{z_2'}+\Gamma_R^{z_1}+\Gamma_R^{z_2}}{2}\right)\rho^{b_s}_{z_1z_2}$$
$$- i\sum_{j=1}^{N}\Omega_j(\rho^{b_s}_{g_r(z_1),z_2} - \rho^{b_s}_{z_1,g_r(z_2)}) + \sqrt{\Gamma_L^{z_1}\Gamma_L^{z_2}}\rho^a_{z_1z_2} + \sqrt{\Gamma_R^{z_1'}\Gamma_R^{z_2'}}\rho^c_{z_1z_2},$$
$$\frac{d\rho^c_{z_1z_2}}{dt} = (i[J_{z_2}-J_{z_1}] - [\Gamma_R^{z_1'}+\Gamma_R^{z_2'}])\rho^c_{z_1z_2}$$
$$-i\sum_{j=1}^{N}\Omega_j(\rho^c_{g_r(z_1),z_2} - \rho^c_{z_1,g_r(z_2)}) + \sqrt{\Gamma_L^{z_1'}\Gamma_L^{z_2'}}(\rho^{b\uparrow}_{z_1z_2} + \rho^{b\downarrow}_{z_1z_2}), \quad (1)$$

where z_1, z_2 indicate qubit states such as $0000, 0001, ..., 1111$ and, $\rho^a_{z_1z_2}$, $\rho^{b_s}_{z_1z_2}$ and $\rho^c_{z_1z_2}$ are density matrix elements when no electron ("a"), one electron ("b") and two electrons ("c") exist in the QPC island, respectively. $J_{0000} = \sum_i^4 \epsilon_i + J_{12} + J_{23}$, $J_{0001} = \sum_i^3 \epsilon_i - \epsilon_4 + J_{12} - J_{23}$, ..., $J_{1111} = -\sum_i^4 \epsilon_i + J_{12} + J_{23}$. $g_l(z_i)$ and $g_r(z_i)$ are introduced for the sake of notational convenience and determined by the relative positions between qubit states as in Ref. [7] (there are 768 equations).

Qubit states, that is, positions of the excess charge in the qubit, influence the QPC tunneling rate as $\Gamma_i^{(\pm)} = \Gamma_{i0} \pm \Delta\Gamma_i = \Gamma_0(1 \pm \zeta)$ with the measurement strength $\zeta \equiv \Delta\Gamma/\Gamma_0$ [7]. Then Γ_L and Γ_R are expressed by $\Gamma_L^{-1} = \Gamma_1^{-1} + \Gamma_2^{-1}$ and $\Gamma_R^{-1} = \Gamma_3^{-1} + \Gamma_4^{-1}$ (Γ_α' is a tunneling rate when the island lies in the "c" state). Time-dependent *fidelity* $F(t) \equiv {\rm Tr}[\hat\rho(0)\hat\rho(t)]$ is calculated by tracing over the elements of the reduced DM obtained from Eq. (1).

We use these DM equations to describe up to four qubits, and compare fidelity of a cluster state $|\Psi\rangle_{\rm CS} = (|+\rangle_1|0\rangle_2|+\rangle_3|0\rangle_4 + |+\rangle_1|0\rangle_2|-\rangle_3|1\rangle_4 + |-\rangle_1|1\rangle_2|-\rangle_3|0\rangle_4 + |-\rangle_1|1\rangle_2|+\rangle_3|1\rangle_4)/2$ with that of a DF state $|\Psi\rangle_{\rm DF} = (|1100\rangle - |1001\rangle - |0110\rangle + |0011\rangle)/2$. Cluster states are generated by the one-step generation method in Ref. [2] where the gate bias voltage ϵ_i for the i-th qubit needs to be set at $\epsilon_i = \epsilon_i^{\rm cs} = \sqrt{E_i^2 - \Omega_i^2}$ at time $t_{\rm cs} = \pi(8n_J+1)/(4J)$. Here, $E_i = (\epsilon_i + \Omega_i^2/\epsilon_i)\cos\alpha_i$ and we

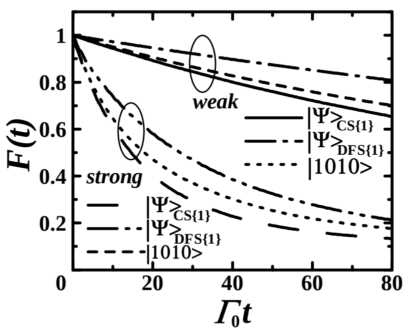

FIG. 2: Time-dependent fidelity of four-qubit states for a cluster state, a DF state, and a product state $|1010\rangle$, in case $\{1\}$. $\Gamma_0 = J$, $\Gamma_i' = \Gamma_i$. The 'weak' indicates a weak measurement case of $\zeta = 0.2$ and the 'strong' indicates a strong measurement case of $\zeta = 0.6$.

also need a relation $J(8n_E - \bar{n}_i)/(8n_J + 1) = E_i$ with an arbitrary integer n_J and the number of the nearest qubits $\bar{n}_i$.

In the present one-dimensional qubit array, we have $E_1 = E_4 = (8n_E - 1)J$ and $E_1 = E_4 = (8n_E - 2)J$. Here, we calculate two examples: case $\{1\}$ is $\Omega = 2J$ ($n_E = 1$) and case $\{2\}$ is $\Omega = 4J$ ($n_E = 1$). In case $\{1\}$, we have $\epsilon_1^{\rm cs} = \epsilon_4^{\rm cs} \approx 5.7J$, $\epsilon_2^{\rm cs} = \epsilon_3^{\rm cs} \approx 6.7J$. In case $\{2\}$ we have $\epsilon_1^{\rm cs} = \epsilon_4^{\rm cs} \approx 4.5J$, $\epsilon_2^{\rm cs} = \epsilon_3^{\rm cs} \approx 5.7J$.

III. NUMERICAL CALCULATIONS

Figure 2 shows a time dependent fidelity when the measurement strength is changed. It can be seen that when the measurement strength is sufficiently large ($\zeta = 0.6$), the fidelity of cluster states greatly degrades. This result shows that the cluster state is sensitive to the existence of trap sites.

Figures 3(a)(b) show the combined effects of the island structure and the nonuniformities among qubits when parameters Ω_i, ϵ_i and Γ_i fluctuate by 10%. In both the strong measurement case ((a)$\zeta = 0.6$) and the weak measurement case ((b)$\zeta = 0.2$), the difference between the cluster state and the DF state is small. This is in contrast to the case without local island structure in QPC discussed in Ref. [2], and shows that the island structure (trap site) imposes a larger decoherence environment and would be a major origin of degradation for the entangled states.

In order to see the effect of nonuniformity between qubits in more detail, we calculate a fidelity at time $t = 50\Gamma_0$ as a function of nonuniformity of qubit parameter η (Fig. 4). We can see that the difference is slight for various types of nonuniformities when η is sufficiently small. We can also see that the fidelity of the larger gate bias case $\{1\}$ is degraded slightly more than that of the smaller gate bias case $\{2\}$. Note that this difference is much smaller in cluster states than in DF states, which is a preferable result for one-way computing in solid-state qubits. From Fig. 3 and Fig. 4, we can confirm that the cluster state is robust to nonuniformity, whereas the cluster state is weak against local electronic state (trap state), similarly to the DF state.

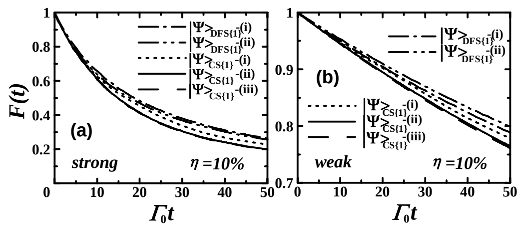

FIG. 3: Time-dependent fidelities $F(t)$ of a cluster state and a DF state when there are nonuniformities in qubit parameter. $\Omega = 4J$ and $n_E = 1$ (case $\{1\}$). $\Gamma_0 = J$, $\Gamma_i' = \Gamma_i$, and $\zeta = 0.6$. (a) Strong measurement case. (b) Weak measurement case. Nonuniformity in the qubit parameters is introduced as $\Omega_i = 2J(1-\eta_i)$, $\epsilon_i = \epsilon + \eta_i J$ and $\Gamma_i^{(\pm)} = (1-\eta_i)\Gamma^{(\pm)}$, with i indicating the i-th qubit. Here $\eta_i = 0$ for all qubits besides $\eta_4 = 0.1$ (i), $\eta_2 = \eta_3 = 0.1$ (ii) and $\eta_4 = 0.1$ (iii). The fidelities of $|\Psi\rangle_{\rm CS}$ for (i) and (ii) mostly overlap.

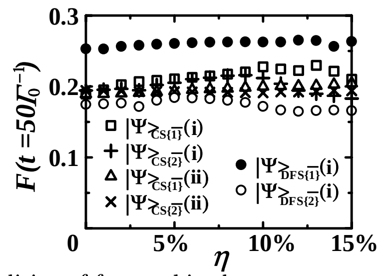

FIG. 4: Fidelities of four-qubit cluster state and DF state at $t = 50\Gamma_0^{-1}$ as a function of nonuniformity η. Parameters are the same as those in Fig. 3.

IV. CONCLUSION

We investigated the effect of local electronic fluctuations (trap state) in addition to nonuniformities, as a decoherence mechanism in cluster states. We found that the island (trap site) affects the fidelity of cluster states significantly. Because the local electronic state provides *dynamical* fluctuation whereas the nonuniformities provide *static* fluctuation, we can say that the cluster states should be careful to *dynamical* fluctuations.

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