Extended Abstracts of the 2011 International Conference on Solid State Devices and Materials, Nagoya, 2011, pp1213-1214

Theory of Resonant Tunneling through a Donor State

Nobuya Mori[1], Amalia Patanè[2], and Laurence Eaves[2]

[1]Division of Electrical, Electronic and Information Engineering, Graduate School of Engineering, Osaka University, Suita, Osaka 565-0871, Japan
Phone: +81-6-6879-7698, E-mail: nobuya.mori@eei.eng.osaka-u.ac.jp
[2]School of Physics & Astronomy, University of Nottingham, Nottingham NG7 2RD, United Kingdom

1 Introduction

The possibility of fine-tuning and imaging the electronic wavefunction in a quantum system is of fundamental interest and has potential for applications in quantum information processing and other advanced technologies [1]. This field is still in its infancy and requires the development of sensitive methods to image electronic wavefunctions. Recently, we used magneto-tunneling spectroscopy (MTS) [2] to probe in situ the spatial compression of a quantum state induced by an applied magnetic field [3]. Here we present a theoretical model for resonant tunneling through a donor state in a resonant tunneling diode under titled magnetic field and compare the results with experiment.

2 Theory

2.1 Resonant Tunneling Diode

We consider a GaAs/AlGaAs resonant tunneling diode whose schematic diagram is given in Fig. 1. We calculate the current due to electron tunneling through the ground state of a Si donor in a GaAs quantum-well (QW) under a titled magnetic field $\boldsymbol{B} = (B_\perp, 0, B_\parallel)$. The component of $\boldsymbol{B}$ parallel to the direction of current, $B_\parallel$, provides us with a means of compressing the donor wavefunction in the QW plane. By measuring the current as a function of the perpendicular component of $B_\perp$, we can probe how the magneto-compression affects the spatial form of the donor wavefunction [3].

2.2 Electronic States

We use a vector potential $\boldsymbol{A} = (-\frac{1}{2}B_\parallel y, \frac{1}{2}B_\parallel x - B_\perp z, 0)$ to express the titled magnetic field. The Hamiltonian for an electron is written as

$$H = \frac{1}{2m^*}(\boldsymbol{p} + e\boldsymbol{A})^2 + V(z) + U(x, y, z). \qquad (1)$$

Here $V(z)$ is the heterostructure confining potential and $U(x, y, z)$ is the additional confining potential due to the Coulomb field of the donor in the QW.

We first solve a 1D wave-equation for the quantized z-motion:

$$\left[-\frac{\hbar^2}{2m^*}\frac{d^2}{dz^2} + V(z)\right]\chi_0(z) = \varepsilon_0\chi_0(z). \qquad (2)$$

We then average the Hamiltonian of Eq. (1) with $\chi_0(z)$ to obtain an effective 2D Hamiltonian $H_{\rm eff} \equiv \langle\chi_0|H|\chi_0\rangle$. In the effective 2D wave-equation of $H_{\rm eff}\Psi(x, y) = E\Psi(x, y)$, we set $\Psi(x, y) = {\rm e}^{ik_y y}\widetilde{\Psi}(x, y)$ with $k_y = eB_\perp\langle z\rangle/\hbar$ to obtain $\widetilde{H}_{\rm eff}\widetilde{\Psi}(x, y) = E\widetilde{\Psi}(x, y)$, where

$$\widetilde{H}_{\rm eff} = \frac{1}{2m^*}(\boldsymbol{p} + e\boldsymbol{A})^2 + E_z(B_\perp) + U^*(x, y), \qquad (3)$$

$E_z(B_\perp) = \varepsilon_0 + (eB_\perp)^2(\langle z^2\rangle - \langle z\rangle^2)/2m^*$, $U^*(x, y) = \langle\chi_0|U(x, y, z)|\chi_0\rangle$, and $\langle z^n\rangle = \langle\chi_0|z^n|\chi_0\rangle$ $(n = 1, 2)$.

For the emitter, the additional Coulomb confinement is weak and we neglect it ($U(x, y, z) = 0$). The lowest Landau state of $\widetilde{H}_{\rm eff}$ is therefore given by

$$\widetilde{\Psi}^m_{\rm e}(x, y) = \frac{1}{\sqrt{2\pi m!}\ell_\parallel}\left(\frac{\rho}{\sqrt{2}\ell_\parallel}\right)^m {\rm e}^{-\frac{\rho^2}{4\ell_\parallel^2}}{\rm e}^{-im\phi}{\rm e}^{i\langle z\rangle_{\rm e} y/\ell_\perp^2}, \qquad (4)$$

where m is the magnetic quantum number ($m = 0, 1, 2, \cdots$), $\rho = (x^2 + y^2)^{1/2}$, $\phi = \tan^{-1}(y/x)$, $\ell_\parallel = (\hbar/eB_\parallel)^{1/2}$, and $\ell_\perp = (\hbar/eB_\perp)^{1/2}$. Note that we have added the subscript 'e', such as $\langle z\rangle_{\rm e}$, to indicate the emitter states. The wavefunction $\Psi(x, y)$ for $H_{\rm eff}$ is then given by $\Psi^m_{\rm e}(x, y) = {\rm e}^{i\langle z\rangle_{\rm e} y/\ell_\perp^2}\widetilde{\Psi}^m_{\rm e}(x, y)$.

For the well, we have an attractive Coulomb potential given by $U(x, y, z) = -e^2/4\pi\epsilon(x^2 + y^2 + z^2)^{1/2}$. We approximate the corresponding effective potential $U^*(x, y)$ to a simple form $U^*(x, y) = -e^2/4\pi\kappa\epsilon\rho$. Here $\epsilon = \epsilon_0\epsilon_{\rm s}$ and κ is an empirical parameter representing the finiteness of the QW width. We then take a variational form of $\widetilde{\Psi}_{\rm w}(x, y) = C{\rm e}^{-\alpha\rho}{\rm e}^{-\beta\rho^2}$ for the donor state in the well. Here α and β are variational parameters determined by minimizing the electron energy, C is a normalization constant, and the subscript 'w' is used for indicating the well states. The wavefunction $\Psi_{\rm w}(x, y)$ is then written as

$$\Psi_{\rm w}(x, y) = C{\rm e}^{i\langle z\rangle_{\rm w} y/\ell_\perp^2}{\rm e}^{-\alpha\rho}{\rm e}^{-\beta\rho^2}. \qquad (5)$$

2.3 Tunneling Matrix Elements

The tunneling matrix elements $\mathcal{M} = \langle \Psi_{\rm w} | \Psi_{\rm e}^{m} \rangle$. Since the emitter states with $m > 0$ do not contribute to the matrix elements $\mathcal{M}$, we have

$$\mathcal{M} = \int_0^\infty J_0(\Delta s \rho / \ell_\perp^2) \phi_{\rm w}(\rho) \phi_{\rm e}(\rho) 2\pi\rho d\rho. \quad (6)$$

Here J_0 is the zeroth order Bessel function of the first kind, $\Delta s = \langle z \rangle_{\rm w} - \langle z \rangle_{\rm e}$ is a tunneling distance, $\phi_{\rm e}(\rho) = {\rm e}^{-\rho^2/4\ell_\parallel^2} / \sqrt{2\pi}\ell_\parallel$, and $\phi_{\rm w}(\rho) = C{\rm e}^{-\alpha\rho}{\rm e}^{-\beta\rho^2}$.

3 Results and Discussion

At $B_\parallel = 0$, we have $\beta = 0$ and the tunneling matrix element $\mathcal{M}$ can be obtained analytically as

$$|\mathcal{M}|^2 = \left(\frac{2}{\alpha\ell_\parallel}\right)^2 \frac{\alpha^6}{(\alpha^2 + \Delta k^2)^3}, \quad (7)$$

with $\Delta k = eB_\perp \Delta s / \hbar$. The normalized tunneling current, $I(B_\perp)/I(0)$, is thus written as

$$\frac{I(B_\perp)}{I(0)} = \frac{\alpha^6}{(\alpha^2 + \Delta k^2)^3} = \frac{B_0^6}{(B_0^2 + B_\perp^2)^3}, \quad (8)$$

with $B_0 = \alpha\hbar / e\Delta s$. We determine B_0 by fitting the measured tunnel current at $B_\parallel = 0$ to Eq. (8). From B_0 we obtain α at $B_\parallel = 0$. The variational parameter α at $B_\parallel = 0$ gives the zero-field donor size λ_0 as $\lambda_0 = 1/\alpha$. Since λ_0 is related to the parameter κ by $\kappa = \lambda_0/(a_{\rm B}/2)$, we can obtain $|\mathcal{M}|^2$ for $B_\parallel > 0$ through variational calculations (see Fig. 2 for the $B_\parallel$ dependence of α and β). The calculated $I(B_\perp)/I(0)$ curves are shown in Fig. 3 and reproduce accurately the measured data. Our model predicts that the $B_\perp$ dependence of the tunnel current becomes weaker at high $B_\parallel$ and changes from approximately polynomial at $B_\parallel = 0$ to Gaussian at high $B_\parallel$, in agreement with experiment (see Figs. 3 and 4 of Ref. [3]).

In conclusion, we have developed a theory for resonant tunneling through a donor state in a tilted magnetic field. We have shown that MTS provides a novel means of probing the magneto-compression of the donor wavefunction. This technique can be extended to probe other types of perturbation on the eigenfunctions in different types of nanostructures.

References

[1] M. Rontani, Nature Materials, **10**, 173 (2011).
[2] P. H. Beton et al., Phys. Rev. Lett., **75**, 1996 (1995).
[3] A. Patanè et al., Phys. Rev. Lett., **105**, 236804 (2010).

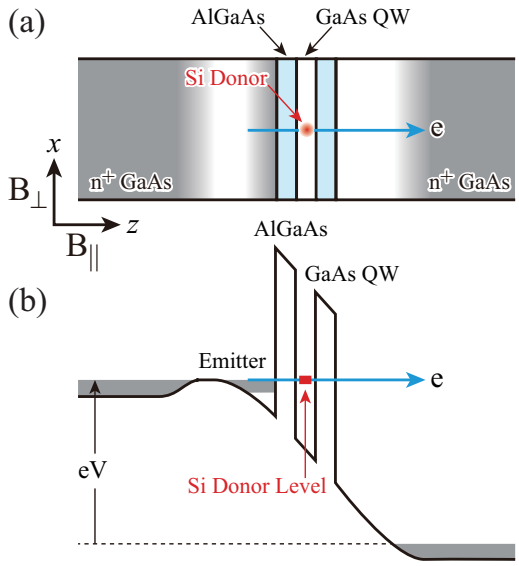

Figure 1: Schematic diagrams of a GaAs/AlGaAs resonant tunneling diode (a) and the conduction-band profile (b).

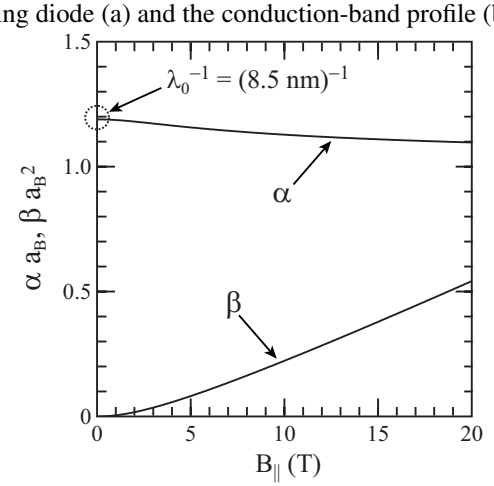

Figure 2: $B_\parallel$ dependence of the variational parameters α and β, normalized by the effective Bohr radius of GaAs, $a_{\rm B}$ (= 10.1 nm).

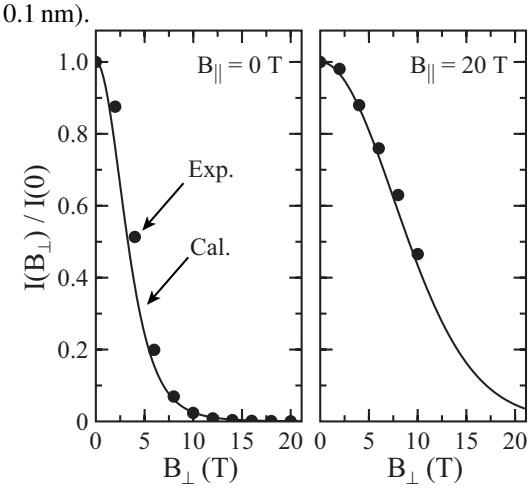

Figure 3: $B_\perp$ dependence of the current at a fixed values of $B_\parallel$. Dots corresponds to the experimental data. Continuous lines are the calculated curves (Δs = 12 nm; λ_0 = 8.5 nm).