Theory of Resonant Tunneling through a Donor State

Nobuya Mori1, Amalia Patane2, and Laurence Eaves2

1Division of Electrical, Electronic and Information Engineering, Graduate School of Engineering, Osaka University, Suita, Osaka 565-0871, Japan
Phone: +81-6-6879-7698, E-mail: nobuya.mori@eei.eng.osaka-u.ac.jp
2School of Physics & Astronomy, University of Nottingham, Nottingham NG7 2RD, United Kingdom

1 Introduction
The possibility of fine-tuning and imaging the electronic wavefunction in a quantum system is of fundamental interest and has potential for applications in quantum information processing and other advanced technologies [1]. This field is still in its infancy and requires the development of sensitive methods to image electronic wavefunctions. Recently, we used magneto-tunneling spectroscopy (MTS) [2] to probe in situ the spatial compression of a quantum state induced by an applied magnetic field [3]. Here we present a theoretical model for resonant tunneling through a donor state in a resonant tunneling diode under tilted magnetic field. The Hamiltonian given by

\[ H = -\frac{\hbar^2}{2m^*} \nabla^2 + V(z) + U(x, y, z). \]  

(1)

where \( V(z) \) is the heterostructure confining potential and \( U(x, y, z) \) is the additional confining potential due to the Coulomb field of the donor in the QW.

We first solve a 1D wave-equation for the quantized \( z \)-motion:

\[ \Psi(x, y, z) = e^{i\Phi(z)} e^{-i\phi} \psi(x, y, z). \]  

(2)

We then average the Hamiltonian of Eq. (1) with \( \langle \Psi | H | \Psi \rangle \) to obtain an effective 2D Hamiltonian \( \tilde{H}_{\text{eff}} \equiv \langle \chi_0 | H | \chi_0 \rangle \).

In the effective 2D wave-equation of \( \tilde{H}_{\text{eff}} \psi(x, y) = E \psi(x, y) \), we set \( \psi(x, y) = e^{ik_0 y} \bar{\psi}(x, y) \) with \( k_0 = eB_L \langle \Psi \rangle / \hbar \) to obtain \( \tilde{H}_{\text{eff}} \psi(x, y) = E \bar{\psi}(x, y) \), where

\[ \tilde{H}_{\text{eff}} = \frac{1}{2m^*} (\rho + eA)^2 + E_z(B_z) + U^*(x, y), \]  

(3)

\[ E_z(B_z) = e_\rho + (eB_z)(\langle \rho^2 \rangle - \langle \rho \rangle^2)/2m^*, \quad U^*(x, y) = \langle \chi_0 | U(x, y, z) | \chi_0 \rangle, \quad \text{and} \quad \langle \rho^2 \rangle = \langle \chi_0 | \rho^2 | \chi_0 \rangle \quad (n = 1, 2). \]

For the emitter, the additional Coulomb confinement is weak and we neglect it \( (U(x, y, z) = 0) \). The lowest Landau state of \( \tilde{H}_{\text{eff}} \) is therefore given by

\[ \tilde{\psi}_e^m(x, y) = \frac{1}{\sqrt{2\pi m!\ell_{\text{||}}}} \left( \frac{\rho}{2\ell_{\text{||}}} \right)^m e^{-\rho^2/4\ell_{\text{||}}} e^{-i\phi} e^{i\ell z/\ell_{\text{||}}}. \]  

(4)

where \( m \) is the magnetic quantum number \( (m = 0, 1, 2, \ldots) \), \( \rho = (x^2 + y^2)^{1/2} \), \( \phi = \tan^{-1}(y/x) \), \( \ell_{\text{||}} = (\hbar/eB_0)^{1/2} \), and \( \ell_{\text{\perp}} = (\hbar/eB_0)^{1/2} \). Note that we have added the subscript ‘e’, such as \( \langle \rho \rangle_e \), to indicate the emitter states. The wavefunction \( \tilde{\psi}_e^m(x, y) \) for \( \tilde{H}_{\text{eff}} \) is then given by

\[ \tilde{\psi}_e^m(x, y) = e^{i\ell z/\ell_{\text{||}}} \tilde{\psi}_e^m(x, y). \]

For the well, we have an attractive Coulomb potential given by \( U(x, y, z) = -e^2/4\pi\epsilon(x^2 + y^2 + z^2)^{1/2} \). We approximate the corresponding effective potential \( U^*(x, y) \) to a simple form \( U^*(x, y) = -e^2/4\pi\kappa e^{i\beta z} \) for the donor state in the well. Here \( \alpha \) and \( \beta \) are variational parameters determined by minimizing the electron energy, \( C \) is a normalization constant, and the subscript ‘w’ is used for indicating the well states. The wavefunction \( \tilde{\psi}_w(x, y) \) is then written as

\[ \tilde{\psi}_w(x, y) = Ce^{i\ell z/\ell_{\text{||}}} e^{-\alpha \rho} e^{-i\beta z}. \]  

(5)
2.3 Tunneling Matrix Elements
The tunneling matrix elements $M = \langle \Psi_w | \Psi_m \rangle$. Since the emitter states with $m > 0$ do not contribute to the matrix elements $M$, we have

$$M = \int_{0}^{\infty} J_0(\Delta s/\ell^2) \phi_w(\rho) \phi_e(\rho) 2\pi d\rho .$$

(6)

Here $J_0$ is the zeroth order Bessel function of the first kind, $\Delta s = \langle z \rangle_w - \langle z \rangle_e$ is a tunneling distance, $\phi_w(\rho) = e^{-\rho^2/4\ell^2} / \sqrt{2\pi \ell}$, and $\phi_e(\rho) = Ce^{-a\rho e^{-b\rho^2}}$.

3 Results and Discussion
At $B_{||} = 0$, we have $\beta = 0$ and the tunneling matrix element $M$ can be obtained analytically as

$$|M|^2 = \left( \frac{2}{\alpha_{||}} \right)^2 \frac{\alpha^6}{(a^2 + \Delta k^2)^3} ,$$

(7)

with $\Delta k = eB_{\perp} \Delta s/h$. The normalized tunneling current, $I(B_{\perp})/I(0)$, is thus written as

$$\frac{I(B_{\perp})}{I(0)} = \frac{\alpha^6}{(a^2 + \Delta k^2)^3} = \frac{B_{0}^6}{(B_{0}^6 + B_{\perp}^2)^3} ,$$

(8)

with $B_0 = a\hbar/e\Delta s$. We determine $B_0$ by fitting the measured tunnel current at $B_{||} = 0$ to Eq. (8). From $B_0$ we obtain $\alpha$ at $B_{||} = 0$. The variational parameter $\alpha$ at $B_{||} = 0$ gives the zero-field donor size $\lambda_0$ as $\lambda_0 = 1/\alpha$. Since $\lambda_0$ is related to the parameter $\kappa$ by $\kappa = \lambda_0/(a_B/2)$, we can obtain $|M|^2$ for $B_{||} > 0$ through variational calculations (see Fig. 2 for the $B_{||}$ dependence of $\alpha$ and $\beta$). The calculated $I(B_{\perp})/I(0)$ curves are shown in Fig. 3 and reproduce accurately the measured data. Our model predicts that the $B_{\perp}$ dependence of the tunnel current becomes weaker at high $B_{||}$ and changes from approximately polynomial at $B_{||} = 0$ to Gaussian at high $B_{||}$, in agreement with experiment (see Figs. 3 and 4 of Ref. [3]).

In conclusion, we have developed a theory for resonant tunneling through a donor state in a tilted magnetic field. We have shown that MTS provides a novel means of probing the magneto-compression of the donor wavefunction. This technique can be extended to probe other types of perturbation on the eigenfunctions in different types of nanostructures.

References