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Steady-state solution for dark states using a three-level system in coupled quantum dots

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1. Introduction

Future quantum communication systems might be composed of optical fiber networks and quantum computers based on solid-state qubits. In this respect, an efficient interconnection between optical and solid-state systems should be developed. Quantum dot (QD) systems, such as GaAs/AlGaAs[1] have discrete energy levels which are suitable for the transfer of photon or phonon energies to electrons in solid-state circuits. Here, we theoretically discuss the interaction between an electromagnetic field and a coupled double QD (DQD) system by focusing on transport properties of three-level system. By adjusting two laser fields, the electron population between the lowest two energy-levels are coherently transferred (coherent population trapping (CPT) or electromagnetically induced transparency (EIT)[2–4]). We derive the density matrix equations in three-level DQD systems, and derive a steady-state solution for a dark state. We investigate a relationship between the time-dependent current characteristic and the dark state.

2. Formulation

A three-energy-level DQD system is realized under a large bias voltage as depicted in Fig.1, in which there is one energy-level (E_1) in the left QD and two (E_2 and E_3) in the right QD. We assume a strong Coulomb blockade regime in which only one excess electron is allowed in each QD. We also assume that the left energy-level E_1 is close to the right upper energy-level E_3 such that electrons in QD 1 tunnel directly into E_3 ($E_3 - E_1 \ll \Omega_L$; and Ω_L is the tunneling rate between QD1 and QD2).

The Hamiltonian is $H = H_0 + H_t + H_l + H_\gamma$, where

$$\begin{aligned} H_0 &= E_1|1\rangle\langle 1| + E_2|2\rangle\langle 2| + E_3|3\rangle\langle 3|, \\ H_t &= -(\Omega_L|1\rangle\langle 3| + \Omega_R e^{-\nu_R t}|2\rangle\langle 3|) + \text{h.c.}, \\ H_l &= \sum_{\alpha=L,R}\sum_{k_\alpha} E_{k_\alpha}|k_\alpha\rangle\langle k_\alpha|, \\ H_\gamma &= \sum_{k_L} V_L|k_L\rangle\langle 1| + \sum_{k_R} V_R|k_R\rangle\langle 2| + \text{h.c.} \end{aligned} \quad (1)$$

Here $|i\rangle$ ($i = 1,2,3$) is the energy-level in QDs, $|k_L\rangle$ ($|k_R\rangle$) is the left(right) electrode state. Ω_R is a transition rate between E_2 and E_3. V_L (V_R) are the tunneling strengths of electrons between the left (right) electrode and the left (right) QD. ν_L and ν_R are electromagnetic fields between E_1 and E_3, and that between E_3 and E_2. The density matrix $\rho_{ij} \equiv |i\rangle\langle j|$ at $T = 0$ is derived by $\dot{\rho} = -\frac{i}{\hbar}[H,\rho] - \frac{1}{2}\{\Gamma,\rho\}$ similarly to Ref.[5] and given by ($\rho_{32} = \tilde{\rho_{32}}e^{-i\nu_R t}$, $\rho_{31} = \tilde{\rho_{31}}e^{-i\nu_L t}$,and $\rho_{21} = \tilde{\rho_{21}}e^{-i[\nu_R-\nu_L]t}$):

$$\dot{\tilde{\rho}}_{00} = \Gamma_1^{(h)}\tilde{\rho}_{11} + \Gamma_2^{(h)}\tilde{\rho}_{22} + \Gamma_3^{(h)}\tilde{\rho}_{33} - [\Gamma_1^{(e)} + \Gamma_2^{(e)} + \Gamma_3^{(e)}]\tilde{\rho}_{00},$$

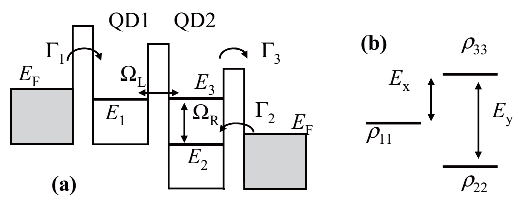

FIG. 1: (a)A three-level system in double QDs (DQDs). A bias voltage is applied between the left and right electrodes. (b) A density matrix for the three level. We define $E_x \equiv E_3 - E_1$ and $E_y \equiv E_3 - E_2$. The dark state is a state with $\rho_{33} = 0$.

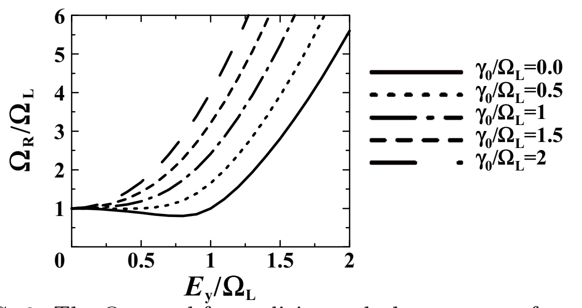

FIG. 2: The Ω_R need for realizing a dark state, as a function of E_y and γ_0. Here, $\gamma_{21} = 0$, $t_0 = 0$ and $\Gamma_1 = \Gamma_2 = \Gamma_3$. These are solutions of $\rho_{33} = 0$ in Eq.(3).

$$\begin{aligned} \dot{\tilde{\rho}}_{11} &= \Gamma_1^{(e)}\tilde{\rho}_{00} - \Gamma_1^{(h)}\tilde{\rho}_{11} + i\Omega_L^*\tilde{\rho}_{31} - i\Omega_L\tilde{\rho}_{13} + it_0(\tilde{\rho}_{12} - \tilde{\rho}_{21}), \\ \dot{\tilde{\rho}}_{22} &= \Gamma_2^{(e)}\tilde{\rho}_{00} - \Gamma_2^{(h)}\tilde{\rho}_{22} + i\Omega_R^*\tilde{\rho}_{32} - i\Omega_R\tilde{\rho}_{23} - it_0(\tilde{\rho}_{12} - \tilde{\rho}_{21}), \\ \dot{\tilde{\rho}}_{33} &= \Gamma_3^{(e)}\tilde{\rho}_{00} - \Gamma_3^{(h)}\tilde{\rho}_{33} + i\Omega_L\tilde{\rho}_{13} - i\Omega_L^*\tilde{\rho}_{31} + i\Omega_R\tilde{\rho}_{23} - i\Omega_R^*\tilde{\rho}_{32}, \\ \dot{\tilde{\rho}}_{31} &= -(i[\omega_{31} - \nu_L] + \gamma_{31})\tilde{\rho}_{31} - i\Omega_L(\tilde{\rho}_{33} - \tilde{\rho}_{11}) + i\Omega_R\tilde{\rho}_{21}, \\ \dot{\tilde{\rho}}_{32} &= -(i[\omega_{32} - \nu_R + \nu_L] + \gamma_{32})\tilde{\rho}_{32} - i\Omega_R(\tilde{\rho}_{33} - \tilde{\rho}_{22}) + i\Omega_L\tilde{\rho}_{12}, \\ \dot{\tilde{\rho}}_{21} &= -(i[\omega_{21} + \nu_R] + \gamma_{21})\tilde{\rho}_{21} - i\Omega_L\tilde{\rho}_{23} + i\Omega_R^*\tilde{\rho}_{31} \\ &\quad - it_0(\tilde{\rho}_{11} - \tilde{\rho}_{22}), \end{aligned} \quad (2)$$

where γ_{31}, γ_{32} and γ_{21} include decoherence such as acoustic phonons. ρ_{i0} and ρ_{0i} are separated and solved analytically but these are irrelevant to the equations regarding transport properties. Γ_i^e represents an electron tunneling from the DQD to the electrodes and $\Gamma_i^{(h)}$ represents that from the electrodes to the DQD. Depending on the relative positions of E_1, E_2 and E_3, we can classify the electron transport. In this paper we consider the case shown in Fig. 1 and set $\Gamma_2^{(e)} = 0$, $\Gamma_1^{(h)} = 0$, and $\Gamma_3^{(e)} = 0$ (this is the case in which dark state explicitly exists). Hereafter we consider $\gamma_{31} = \gamma_0 = \gamma_{32}$ and $\nu_L = 0$.

3. Steady-state solutions for the dark state

Steady-state solutions are obtained from the density matrix equations when $d\rho_{ij}/dt = 0$. Compared with the optical three-level[6], the existence of the ρ_{00} state complicates the equations. Dark state corresponds to the case where there is no electron state in E_3 as $\tilde{\rho}_{33} = 0$. When $t_0 = 0$, we can express the steady-state solution by fourth-order polynomial equations of E_x, E_y and Ω_R, such as

$$\rho_{33} \propto A_{33}(\Omega_R/\Omega_L)^4 + B_{33}(\Omega_R/\Omega_L)^2 + C_{33} = 0, \quad (3)$$

where

$$\begin{aligned} A_{33} &= \gamma_0'(-\Gamma_2' + \gamma_{21}'), & (4) \\ B_{33} &= -E_y' D_z' \Gamma_2' \gamma_{21}' + D_z^{2\prime}(\Gamma_2' + \gamma_0')\gamma_0' \\ &+ \gamma_{21}'\gamma_0' + \gamma_{21}^{2\prime}\gamma_0^{2\prime}, & (5) \\ C_{33} &= \Gamma_2'\gamma_0'((1 + D_z' E_y')^2 + \gamma_{21}^{2\prime} E_y^{2\prime} + (D_z^{2\prime} + \gamma_{21}^{2\prime})\gamma_0^{2\prime} \\ &+ 2\gamma_{21}'\gamma_0'), & (6) \end{aligned}$$

with $D_z \equiv E_x - E_y$ (all quantities are rescaled by Ω_L and indicated by the prime symbol, such as $\Gamma_2' = \Gamma_2/\Omega_L$). This equation is a parabolic function regarding Ω_R^2 with $C_{33} > 0$. Thus, if $\Gamma_2 > \gamma_{21}$, $\rho_{33}(\Omega_R^2) = 0$ has a solution for positive Ω_R^2. Also when $\Gamma_2 > \gamma_{21}$, the coefficient of Ω_R^4 has a negative value, therefore, the Ω_R^2 of Eq.(3) for the dark state is a maximum value for the solutions of the density matrix equations to be valid. Figure 2 plots Ω_R, which satisfies $\rho_{33}(\Omega_R^2) = 0$ as a function of E_y/Ω_L. Note that, when E_y increases, Ω_R should increase.

4. Time-dependent current

Here, we show numerical results of the time-dependent matrix element ρ_{33} and current. Ω_R is calculated from Eq. (3) such that the initial E_y is given, *e.g.*, as $E_y/\Omega_L = 2$. Figure 3 shows the time-dependent density matrix element ρ_{33} when $E_x = 0$, $t_0 = 0$ and $\gamma_{21} = 0$ starting from (a) $|1\rangle$ and (b) $(|1\rangle + |2\rangle)/2$. It can be seen that, as E_y decreases, $\rho_{33}(t)$ decreases. As mentioned above, Ω_R is determined such that it satisfies the steady-state solution $\rho_{33}(t \to \infty) \to 0$ for $E_y/\Omega_L = 2$ where $\rho_{33}(t \to \infty)$ has the lowest values. Compared with Fig 3(a), Figure 3(b) oscillates faster. This is because for the superposition state, the density population of electrons oscillates between $|1\rangle$ and $|2\rangle$ more often than the case starting from $|1\rangle$.

Figure 4 show the time-dependent currents through the DQD system. The current is derived[7] as $I(t) = \Gamma_R(\rho_{33}(t) + \rho_{22}(t))$. Here we consider $\tilde{I}(t) \equiv e^{i\nu_R t} I(t)$. Ω_R is determined similarly to Fig 3. Thus current is expected to be reduced at $E_y/\Omega_L = 2$. Figures 4(a,c) shows that current decreases around the expected dark state. Figures 4(c, d) show time-dependent currents starting from a superposition state of $(|1\rangle + |2\rangle)/2$. Compared with Figs. 4(a, c), Figs 4(b, d) show that a finite leak tunneling ($t_0 = 0.5$ and $\gamma_{21} = 0.5$) leads to a small current reduction, and the evidence of the dark state disappears regardless of the initial state.

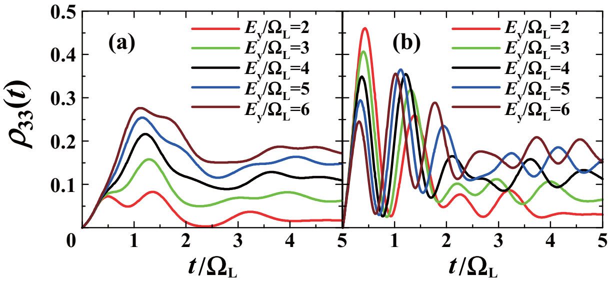

FIG. 3: Time-dependent density matrix elements $\rho_{33}(t)$ starting from an initial state of (a) $|1\rangle$ and (b) $(|1\rangle + |2\rangle)/2$. Here, $\gamma_0/\Omega_L = 1$, $t_0 = 0$ and $\gamma_{21} = 0$. Also, $E_y/\Omega_L = 5$ corresponds to the solution of Eq.(3).

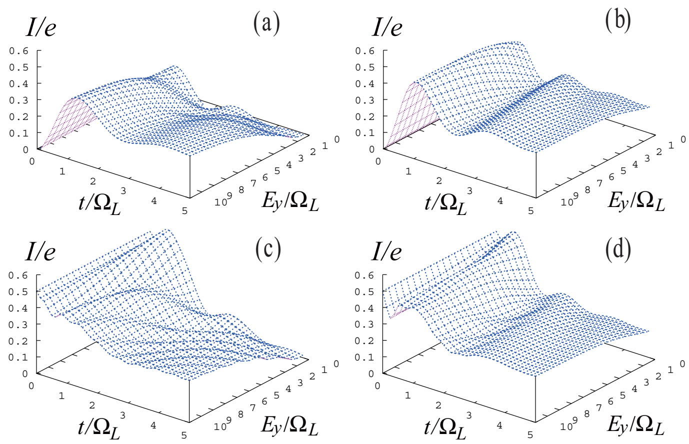

FIG. 4: Time-dependent current as a function of E_y starting from $|1\rangle$ for (a) and (b), $(|1\rangle + |2\rangle)/2$ for (c) and (d). $\Gamma_1/\Omega_L = \Gamma_2/\Omega_L = \Gamma_3/\Omega_L =$, $\gamma_0/\Omega_L = 1$, (a)(c) $t_0/\Omega_L = 0$ and $\gamma_{21}/\Omega_L = 0$. (b)(d) $t_0/\Omega_L = 1$ and $\gamma_{21}/\Omega_L = 1$.

5. Conclusion

We theoretically solved the steady-state solutions of the density matrix equations for a thee-level DQD system, and showed the condition of the appearance of a dark state. Numerical calculations for time-dependent current characteristics showed that the steady-state can be detected by measuring current.

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