Steady-state solution for dark states using a three-level system in coupled quantum dots

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1. Introduction
Future quantum communication systems might be composed of optical fiber networks and quantum computers based on solid-state qubits. In this respect, an efficient interconnection between optical and solid-state systems should be developed. Quantum dot (QD) systems, such as GaAs/AlGaAs1 have discrete energy levels which are suitable for the transfer of photon or phonon energies to electrons in solid-state circuits. Here, we theoretically discuss the interaction between an electromagnetic field and a coupled double QD (DQD) system by focusing on transport properties of three-level system. By adjusting two laser fields, the electron population between the lowest two energy-levels are coherently transferred (coherent population trapping (CPT)) or electromagnetically induced transparency (EIT).2 We derive the density matrix equations in three-level DQD systems, and derive a steady-state solution for a dark state. We investigate a relationship between the time-dependent current characteristic and the dark state.

2. Formulation
A three-energy-level DQD system is realized under a large bias voltage as depicted in Fig.1, in which there is one energy-level (E1) in the left QD and two (E2 and E3) in the right QD. We assume a strong Coulomb blockade regime in which each QD only contains one electron, and then we also assume that the left energy-level E1 is close to the right upper energy level E2 such that QD1 and QD2.

The Hamiltonian is \( H = H_0 + H_1 + H_1 + H_7 \), where
\[
H_0 = E_{11}[1]\{1\} + E_{22}[2\{2\} + E_{33}\{3\},
\]
\[
H_1 = -(\Omega_1|\{3\} + i\Omega_2 e^{-i\pi/2}|\{2\}) + \text{h.c.},
\]
\[
H_7 = \sum_{a=L, R} \sum_{k_0} E_{k_0}|k_{a}\rangle\langle k_{a}| + \text{h.c.},
\]
\[
H_7 = \sum_{k_0} V_{k_0}|k_{L}\rangle\langle 3\} + \sum_{k_0} V_{k_0}|k_{R}\rangle\langle 2\} + \text{h.c.} \quad (1)
\]

Here \(|i\rangle (i = 1, 2, 3)\) is the energy level in QDs, \(|k_{L}\rangle (|k_{R}\rangle)\) is the left(right) electrode state. \(\Omega_1, \Omega_2\) are the tunneling rate between the left and right QDs. \(V_{k_0}\) are the tunneling strengths of electrons between the left (right) electrode and the left (right) QD. \(\nu_{L}\) and \(\nu_{R}\) are the electromagnetic fields between E1 and E3, and that between E3 and E2. The density matrix \(\rho_{ij} \equiv \langle i|\{j\}\rangle\) at \(T = 0\) is derived by \(\dot{\rho} = -i[H, \rho] - \frac{1}{\hbar}(\Gamma_{\rho}, \rho)\) similarly to Ref.5 and given by (\(\rho_{22} = \rho_{22} e^{-i\nu_{RT}}, \rho_{33} = \rho_{33} e^{-i\nu_{RT}}\), and \(\rho_{23} = \rho_{23} e^{-i\nu_{RT}}\)).
\[
\dot{\rho}_{11} = \Gamma_1^{(e)} \rho_{11} + \Gamma_1^{(h)} \rho_{11} + i\Omega_{L}\rho_{31} - i\Omega_{R}\rho_{31} + i\Omega_{L}\rho_{31} + i\Omega_{R}\rho_{31} - \delta_{12} \rho_{12} - \rho_{21},
\]
\[
\dot{\rho}_{22} = \Gamma_2^{(e)} \rho_{22} + \Gamma_2^{(h)} \rho_{22} + i\Omega_{L}\rho_{32} - i\Omega_{R}\rho_{32} - i\Omega_{L}\rho_{32} - i\Omega_{R}\rho_{32},
\]
\[
\dot{\rho}_{33} = \Gamma_3^{(e)} \rho_{33} + \Gamma_3^{(h)} \rho_{33} + i\Omega_{L}\rho_{33} - i\Omega_{R}\rho_{33} + \Gamma_3^{(e)} \rho_{33} - \nu_{L} \rho_{33} - \nu_{R} \rho_{33},
\]
\[
\dot{\rho}_{23} = \rho_{23} \rho_{12} - \rho_{21} \rho_{12},
\]
\[
\dot{\rho}_{21} = \rho_{21} \rho_{12} - \rho_{23} \rho_{31}.
\]

\(\Omega_1, \Omega_2\) include decoherence such as acoustic phonons. \(\rho_{23}\) and \(\rho_{31}\) are separated and solved analytically but these are irrelevant to the equations regarding transport properties. \(\Gamma^{(e)}\) represents an electron tunneling from the DQD to the electrodes and \(\Gamma^{(h)}\) represents that from the electrodes to the DQD. Depending on the relative positions of E1, E2 and E3, we can classify the electron transport. In this paper we consider the case shown in Fig.1 and set \(\Gamma^{(e)} = 0\), \(\Gamma^{(h)} = 0\), and \(\Gamma^{(e)} = 0\) (this is the case in which dark state explicitly exists). Hereafter we consider \(\nu_{L} = 0\).

FIG. 1: (a) A three-level system in double QDs (DQDs). A bias voltage is applied between the left and right electrodes. (b) A density matrix for the three levels. We define \(E_s \equiv E_3 - E_1\) and \(E_p \equiv E_3 - E_2\). The dark state is a state with \(\rho_{33} = 0\).

\[
\gamma_1^{(e)} = 0.0, \quad \gamma_1^{(h)} = 0.5, \quad \gamma_1 = 1.0, \quad \gamma_2 = 2.0
\]

FIG. 2: The \(\Omega_1\) need for realizing a dark state, as a function of \(\nu_1\) and \(\gamma_0\). Here, \(\gamma_1 = 0, \nu_1 = 0\) and \(\Gamma_1 = \Gamma_2 = \Gamma_3\). These are solutions of \(\rho_{33} = 0\) in Eq.(3).
3. Steady-state solutions for the dark state

Steady-state solutions are obtained from the density matrix equations when \(dp_{ij}/dt = 0\). Compared with the optical three-level\(^4\), the existence of the \(\rho_{00}\) state complicates the equations. Dark state corresponds to the case where there is no electron state in \(E_3\) as \(\rho_{33} = 0\). When \(t_0 = 0\), we can express the steady-state solution by fourth-order polynomial equations of \(E_x, E_y\) and \(\Omega_R\), such as

\[
\rho_{33} \propto A_{33}(\Omega_R/\Omega_L)^4 + B_{33}(\Omega_R/\Omega_L)^2 + C_{33} = 0, \quad (3)
\]

where

\[
A_{33} = \gamma_0(-\Gamma_2^2 + \gamma_2^2), \quad (4)
\]

\[
B_{33} = -E_y^2\Gamma_2^4\gamma_2^2 + D_{2}^2 \Gamma_2^2 (\Gamma_2^2 + \gamma_0^2)\gamma_0^2 + \gamma_2^2\gamma_0^2 + \gamma_2^2\gamma_0^2, \quad (5)
\]

\[
C_{33} = \Gamma_2^2\gamma_0^2((1 + D_{2}^2 E_y^2)^2 + \gamma_2^2 E_y^2) + (D_{2}^2 + \gamma_2^2)^2 \gamma_0^2 + 2\gamma_2^2 \gamma_0^2, \quad (6)
\]

with \(D_2 \equiv E_x - E_y\) (all quantities are rescaled by \(\Omega_L\) and indicated by the prime symbol, such as \(\Gamma_2^2 = \Gamma_2/\Omega_L\)). This equation is a parabolic function regarding \(\Omega_R^2\) with \(C_{33} > 0\). Thus, if \(\Gamma_2 > \gamma_2\), \(\rho_{33}(\Omega_R^2) = 0\) has a solution for positive \(\Omega_R^2\). Also when \(\Gamma_2 > \gamma_2\), the coefficient of \(\Omega_R^2\) has a negative value, therefore the \(\Omega_R^2\) of Eq.(3) for the dark state is a maximum value for the solutions of the density matrix equations to be valid. Figure 2 plots \(\Omega_R\), which satisfies \(\rho_{33}(\Omega_R^2) = 0\) as a function of \(E_y/\Omega_L\). Note that, when \(E_y\) increases, \(\Omega_R\) should increase.

4. Time-dependent current

Here, we show numerical results of the time-dependent matrix element \(\rho_{33}\) and current. \(\Omega_R\) is calculated from Eq. (3) such that the initial \(E_y\) is given, e.g., as \(E_y/\Omega_L = 2\). Figure 3 shows the time-dependent density matrix element \(\rho_{33}\) when \(E_y = 0, t_0 = 0\) and \(\gamma_2 = 0\) starting from \(|1\rangle\) and \(|1\rangle (|1\rangle + |2\rangle)/2\). It can be seen that, as \(E_y\) decreases, \(\rho_{33}(t)\) decreases. As mentioned above, \(\Omega_R\) is determined such that it satisfies the steady-state solution \(\rho_{33}(t \to \infty)\to 0\) for \(E_y/\Omega_L = 2\) where \(\rho_{33}(t \to \infty)\) has the lowest values. Compared with Fig 3(a), Figure 3(b) oscillates faster. This is because for the superposition state, the density population of electrons oscillates between \(|1\rangle\) and \(|2\rangle\) more often than the case starting from \(|1\rangle\).

Figure 4 shows the time-dependent currents through the DQD system. The current is derived as \(I(t) = \Gamma_R(\rho_{31}(t) + \rho_{21}(t))\). Here we consider \(I(t) \equiv e^{i\omega t} I(t)\), \(\Omega_R\) is determined similarly to Fig 3. Thus current is expected to be reduced at \(E_y/\Omega_L = 2\). Figures 4(c, d) show time-dependent currents starting from a superposition state of \((|1\rangle + |2\rangle)/2\). Compared with Figs. 4(a, c), Figs 4(b, d) show that a finite leak tunneling \((t_0 = 0.5\) and \(\gamma_2 = 0.5\) leads to a small current reduction, and the evidence of the dark state disappears regardless of the initial state.

![Figure 3](image-url)

**FIG. 3:** Time-dependent density matrix elements \(\rho_{33}(t)\) starting from an initial state of (a) \(|1\rangle\) and (b) \((|1\rangle + |2\rangle)/2\). Here, \(\gamma_0/\Omega_L = 1, t_0 = 0\) and \(\gamma_2 = 0.5\). Also, \(E_y/\Omega_L = 5\) corresponds to the solution of Eq.(3).

![Figure 4](image-url)

**FIG. 4:** Time-dependent current as a function of \(E_y\) starting from (a) \(|1\rangle\) and (b) \((|1\rangle + |2\rangle)/2\) for (c) and (d). \(\Gamma_1/\Omega_L = \Gamma_2/\Omega_L = \Gamma_3/\Omega_L = \gamma_0/\Omega_L = 1\), (a)(c) \(t_0/\Omega_L = 0\) and \(\gamma_2/\Omega_L = 0\). (b)(d) \(t_0/\Omega_L = 1\) and \(\gamma_2/\Omega_L = 1\).

5. Conclusion

We theoretically solved the steady-state solutions of the density matrix equations for a three-level DQD system, and showed the condition of the appearance of a dark state. Numerical calculations for time-dependent current characteristics showed that the steady-state can be detected by measuring current.

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