Analytical Formulas for the Drain Current of Silicon Nanowire MOSFET

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1. Introduction

Silicon nanowire(NW) MOSFETs attract wide attention as a promising nanodevice for future high-density LSI application. For development of the device including the circuit application, a handy tool that affords accurate prediction of device characteristics is indispensable. So far, a drain current equation based on the kT-layer theory has been proposed by Lundstrom's group[1]. It is quite handy, but is limited to the saturation current of the device. The simple expression sometimes causes argument about accuracy. We also proposed a compact model of the device current[2,3]. But it still requires numerical calculation by a personal computer. No reliable tool handled by a calculator was provided as yet.

This paper proposes two analytical formulas for the drain current of silicon nanowire nMOSFET. They allow evaluation of device characteristics by means of handy calculators, and are useful for approximate evaluation and investigation of device characteristics.

2. Analytic Expressions of the Drain Current

We have already reported a compact model of Si quasi-ballistic NW MOSFET[2]. The scattering model in Fig. 2 is employed in the model and considers the elastic scattering and the energy relaxation due to optical phonon emission. The drain current is readily evaluated by following a simple procedure if device parameters and bias conditions are specified. A comparison with a reported result of detailed numerical simulation[4] showed good agreement, indicating the usefulness of the compact model.

The compact model includes the energy integration as well as the subband summation, and numerical procedures are inevitable for the detailed evaluation. But here, we introduce some reasonable approximations in the NW structure and the carrier statistics in the compact model, and intend to derive analytic expressions of the device current.

As for the modeled device structure, we assume a comparatively thick nanowire MOSFET where the volume inversion is negligible. We can assume such a device as the practical NW MOSFET in near future, of which a sufficient current-drivability is required in the circuit application. Then the inversion layer of the MOSFET tends to be located on the surface of NW as is illustrated in Fig. 3. It has a ring structure and the wave function as well as the energy

subband within the layer is compactly approximated as in the figure, if higher subbands in *r*-direction are neglected. The subband summation is simplified, and it is replaced by an integration.

The carrier distribution obeys Fermi statistics, but the integration over energy is formidable for the numerical evaluation. According to our result in ballistic MOSFET[5], the variation of drain current when they are replaced by the fully degenerate carrier distribution (step-function type) is generally small. The large temperature dependence observed in the device current is due to the large temperature dependence in carrier scattering. It is a practical approximation to employ the step-function type distribution for carrier statistics, and the carrier scattering is treated differently. The carrier scattering in our model is energy-dependent represented by an transmission probability, and the expression includes scattering parameters that can be adjusted to be consistent with experimental results. If the transmission probability is replaced by an energy-averaged value and is taken out of the integration, the energy integration can be conducted analytically. Then we can derive the first formula of drain current shown in Fig. 4.

On the other hand, the device characteristics are believed to be described by Boltzmann statistics if the carrier degeneracy in the device is not conspicuous, e.g. at low $V_{\rm G}$ values. Boltzmann statistics also allow derivation of the analytical drain current, when the similar assumptions are made. The derived second formula is shown in Fig. 5.

Notice that the two formulas in Figs. 4 and 5 are neither independent nor parallel to each other. They correspond to two incompatible cases; one with completely degenerate carriers and the other controlled by Boltzmann statistics. They rather represent two complementary aspects of the same device. For sufficiently large V_D values in the second formula, one can see that the expression is reduced to the Lundstrom's formula. But the second formula itself covers the both drain currents in the linear region and the saturation region.

The $I-V_D$ characteristics in a sample device (NW Diameter=10 nm, L=20 nm, $t_{oxide}=1$ nm) are compared for the original compact model (shown in Fig. 6), the first formula (Fig. 7) and the second formula (Fig. 8). Agreement is satisfactory in view of the simple expression of analytic

formulas.

3. Summary

Analytical formulas for the drain current of silicon NW nMOSFETs are proposed. They allow evaluation of device characteristics using handy calculators. The result is in satisfactory agreement with the more detailed compact model, suggesting utility of the formulas as approximation.



Fig. 1. NW MOSFET structure.

Thick NW Modeling



Summation over subband is relplaced by an integration.

Fig. 3. A thick NW is assumed, and compact expressions for the electronic states and the subband energy are adopted.

Drain Current (2) (Boltzmann Statistics Model)

$$\frac{1}{C_G} = \frac{1}{C_{ox}} + \frac{1}{C_{inv}} + \frac{1}{C_Q} \qquad v_{inj} = \sqrt{\frac{2kT}{\pi m_x}}$$

$$R = 1 - \frac{2qV_D}{\left(\sqrt{B_0 + D_0} + \sqrt{D_0}\right)qV_D + \sqrt{2m_x D_0}B_0L\ln(3.15)}$$

$$I_D = C_G(V_G - V_t)v_{inj} \frac{(1 - R)\left\{1 - \exp\left(-\frac{qV_D}{kT}\right)\right\}}{(1 + R) + (1 - R)\exp\left(-\frac{qV_D}{kT}\right)}$$

Fig. 5. The second formula for the drain current of a NW nMOSFET based on the Boltzmann statistics.



Fig. 7. I-V_D plot by the first formula in Fig. 4. The fully degenerate carrier model.

Reference

- [1] M. Lundstrom, IEEE Elec. Dev. Lett., pp.361, (1997).
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- [3] K. Natori, Ext. Abst. Intern. Conf. SSDM, pp.1058, (2009).
- [4] S. Jin et al., IEEE Trans. Elec. Dev., vol.55, pp.727, (2008).
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quasi-ballistic NW MOSFET.

Drain Current (1) (Degenerate Carrier Model)

$$\begin{aligned} \frac{1}{C_{eff}} &= \frac{1}{C_{ox}} + \frac{1}{C_{inv}}, & C_q = q^2 \frac{m}{\pi \hbar^2} (2\pi r_0) \\ T &= \frac{2qV_D}{\left(\sqrt{B_0 + D_0 + \sqrt{D_0}}\right) qV_D + \sqrt{2mD_0} B_0 L \ln(3.15)} \\ V_{Dsat} &= \frac{C_{eff} (V_G - V_t)}{\alpha C_{eff} + 2C_q (2 - T)}, & \frac{\mu_s}{q} = \frac{C_{eff} (V_G - V_t) + 2C_q T V_D}{\alpha C_{eff} + 4C_q}, \\ V_D &\leq V_{Dsat}, & I_D = \frac{8}{3\pi q} \sqrt{\frac{2}{m}} C_q T \left[\mu_s^{3/2} - (\mu_s - qV_D)^{3/2} \right] \\ V_{Dsat} &\leq V_D, & I_D = \frac{8}{3\pi q} \sqrt{\frac{2}{m}} C_q T \left(qV_{Dsat} \right)^{3/2} \end{aligned}$$

Fig. 4. The first formula for the drain current of a NW nMOSFET based on the fully degenerate carrier model.



Fig. 6. I-V_D plot by the quasi-ballistic compact model previously proposed.



Fig. 8. I- V_D plot by the second formula in Fig. 5. The Boltzmann statistics model.