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Detection of a charge qubit by the Kondo and the Fano-Kondo effects in quantum dots

Tetsufumi Tanamoto[1], Yu-xi Liu[2], Xuedong Hu[3] and Franco Nori[4]
[1]Toshiba R & D Center, Kawasaki, Japan, [2]Tsinghua University, Beijing, China, [3]University at Buffalo, SUNY, USA, [4]RIKEN, Japan, [4]The University of Michigan, USA,
[1]Phone:+81-44-549-2192, E-mail:tetsufumi.tanamoto@toshiba.co.jp

1. Introduction

The Kondo effect and the Fano-Kondo effect are important phenomena that have been observed in quantum dots (QDs)[1-5]. We theoretically investigate the transport properties of a coupled QD system [6](Fig.1) in order to study the possibility of detecting a qubit state from the modulation of the conductance peak in the Kondo effect and the dip in the Fano-Kondo effect. We use a slave-boson mean-field theory (SBMFT)[3-5] with the help of nonequilibrium Keldysh Green functions. We show that the peak and dip of the conductance are both shifted depending on the qubit state, and that we can estimate the optimal point and tunneling coupling between the two qubit eigenstates.

2. Formulation

The Hamiltonian has three terms $H = H_{\rm det} + H_{\rm q} + H_{\rm int}$, where $H_{\rm det}$ describes the detector, $H_{\rm q}$ the charge qubit, and $H_{\rm int}$ the interaction between the charge qubit and the detector. Here $H_{\rm q}$ is written as $H_{\rm q} = \Omega(d_a^\dagger d_b + d_b^\dagger d_a) + \varepsilon_q(d_a^\dagger d_a - d_b^\dagger d_b)$, where d_a and d_b are electron annihilation operators of the upper QD a and the lower QD b in the charge qubit, respectively. The Kondo (K) detector Hamiltonian is $H_{\rm det}^{\rm (K)} = H_{\rm SD} + H_{\rm QD}^{\rm (K)}$, and the Fano-Kondo (F) detector Hamiltonian is $H_{\rm det}^{\rm (F)} = H_{\rm SD} + H_{\rm QD}^{\rm (F)}$, [5] where

$$H_{\rm SD} = \sum_{\alpha=L,R}\sum_{k_\alpha,s}\{\varepsilon_{k_\alpha} c_{k_\alpha s}^\dagger c_{k_\alpha s} + V_\alpha(c_{k_\alpha s}^\dagger f_{ds} + f_{ds}^\dagger c_{k_\alpha s})\}, \quad (1)$$

$$H_{\rm QD}^{\rm (K)} = \sum_s \varepsilon_d f_{ds}^\dagger f_{ds} + \lambda_d\left[\sum_s f_{ds}^\dagger f_{ds} + b_d^\dagger b_d - 1\right], \quad (2)$$

$$\begin{aligned} H_{\rm QD}^{\rm (F)} &= \sum_{\alpha_1=c,d}\sum_s \varepsilon_{\alpha_1} f_{\alpha_1 s}^\dagger f_{\alpha_1 s} + \lambda_c\left[\sum_s f_{cs}^\dagger f_{cs} + b_c^\dagger b_c - 1\right] \\ &+ t_d\sum_s(f_{ds}^\dagger b_c^\dagger f_{cs} + f_{cs}^\dagger b_c f_{ds}). \end{aligned} \quad (3)$$

Here ε_{k_α} is the energy level for electrodes ($\alpha = L, R$); ε_c and ε_d are energy levels for the two QDs, respectively. $c_{k_\alpha s}$ and $f_{\alpha_1 s}$ are annihilation operators of the electrodes, and of the QDs ($\alpha_1 = c, d$), respectively; s is the spin degree of freedom with spin degeneracy 2; We take $\langle b_{\alpha_1}\rangle$ and $\tilde{\varepsilon}_{\alpha_1} \equiv \varepsilon_{\alpha_1} + \lambda_{\alpha_1}$ as mean-field parameters.

$H_{\rm int}$ is derived from a capacitance network model[6] such as $H_{\rm int} = V_q z_{\alpha_1}\ \sigma^z n_{\alpha_1}$, where n_c and n_d are the numbers of electrons in the trap QD c and QD d. σ^z is given by $\sigma_z = d_a^\dagger d_a - d_b^\dagger d_b$. By using the decoupling approximation [7], we have

$$H_{\rm int}^{\rm MF} \equiv V_q z_{\alpha_1}\{\langle\sigma^z\rangle n_{\alpha_1} + \sigma^z\langle n_{\alpha_1}\rangle - \langle\sigma^z\rangle\langle n_{\alpha_1}\rangle\} \quad (4)$$

where $\alpha_1 = c, d$. By this decoupling, we have the relations for Fano-Kondo detector:

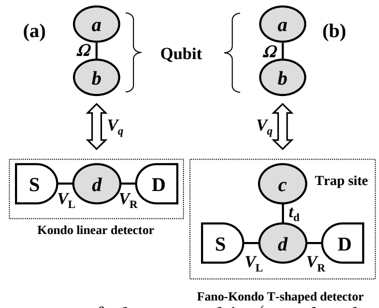

FIG. 1: Two types of charge qubit (two-level system) detectors. (a) The charge qubit detected by the Kondo effect. (b) The charge qubit detected by the Fano-Kondo effect.

$$\varepsilon_c' = \varepsilon_c + \lambda_c + V_q z_c \frac{\varepsilon_q'}{\Delta}\tanh\frac{\beta\Delta}{2}, \quad (5)$$

$$\varepsilon_q' = \varepsilon_q + V_q z_c[1 - z_c], \quad (6)$$

(we have similar relation for the Kondo detector). The detector current is calculated as [5]

$$J = \frac{e}{\hbar}\int\frac{d\omega}{\pi}\frac{z_d^2\Gamma_L\Gamma_R(\omega-\varepsilon_c')^2}{C_{00}}\left[f_L(\omega) - f_R(\omega)\right], \quad (7)$$

with $C_{00} \equiv [(\omega-\varepsilon_c')(\omega-\varepsilon_d) - |\tilde{t}_d|^2]^2 + \gamma^2(\omega-\varepsilon_c')^2$. and $\gamma = (\Gamma_L + \Gamma_R)/2$. $f_L(\omega)$ and $f_R(\omega)$ are the Fermi distribution functions of the electrodes.

3. Numerical results

Figure 2 shows the conductance of the Kondo detector [(a) and (c)] and the Fano-Kondo detector [(b) and (d)]. The peaks and dips are maximized when the coherence between the discrete energy state and the continuum states is largest, we thus denote corresponding energies $\varepsilon_d^{\rm (peak)}$ and $\varepsilon_c^{\rm (dip)}$ as *coherent extrema*. For both detectors, the shifts of the conductance peaks and dips are observed, when ε_q is changed.

As the qubit bias ε_q increases, the distribution of the excess charge in the qubit approaches to the detector QDs, resulting in raising the energy of QD d of the Kondo detector and that of QD c of the Fano-Kondo detector. Finally, the increase of the QD energies are saturated because of the balance of the charge distribution. Figure 3 reflects this fact and shows that $\varepsilon_d^{\rm (peak)}$ and $\varepsilon_c^{\rm (dip)}$ increase as ε_q increase.

Because $z_{\alpha_1} = 0$ ($\alpha_1 = c, d$) is satisfied at the coherent extrema, we have the relation $\varepsilon_q' = \varepsilon_q$ from Eq. (6). Then, it can be observed that the minimum of $\varepsilon_d^{\rm (peak)}$

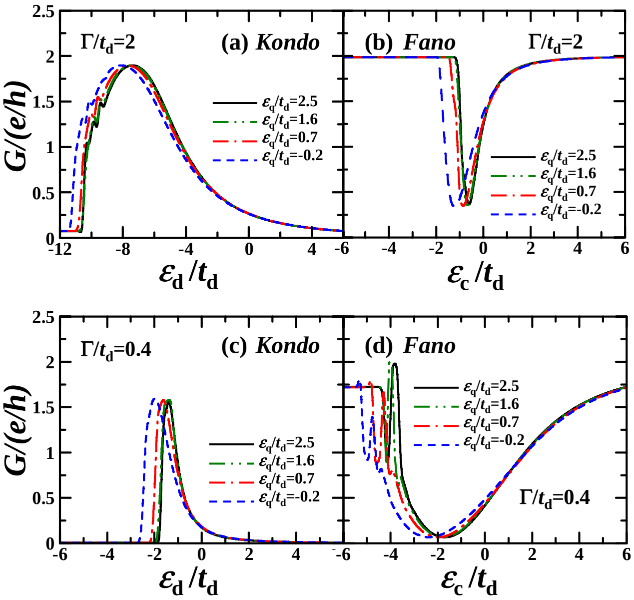

FIG. 2: Numerical results for the conductance G for detectors as a function of the QD energies for $\Omega/t_d = 1$, $V_q/t_d = 0.5$ and temperature $T/t_d = 0.02$. (a) Fast Kondo detector :$\Gamma/t_d = 2$. (b) Fast Fano-Kondo detector :$\Gamma/t_d = 2$. (c) Slow Kondo detector :$\Gamma/t_d = 0.4$. (d) Slow Fano-Kondo detector :$\Gamma/t_d = 0.4$. The peak positions for the Kondo detector and the dip positions for the Fano-Kondo effect are shifted by the qubit bias ε_q.

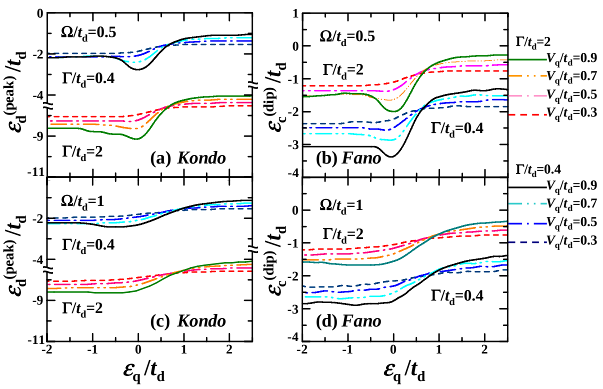

FIG. 3: The coherent extrema $\varepsilon_d^{(\mathrm{peak})}$ (conductance peak) and $\varepsilon_c^{(\mathrm{dip})}$ (conductance dip), as a function of the qubit bias ε_q. The conductance peak of the Kondo detector for the $\Omega/t_d = 0.5$ qubit (a), and the $\Omega/t_d = 1$ qubit (c). The conductance dip of the Fano-Kondo detector for the $\Omega/t_d = 0.5$ qubit (b), and the $\Omega/t_d = 1$ qubit (d). The $\varepsilon_d^{(\mathrm{peak})}$ and $\varepsilon_c^{(\mathrm{dip})}$ are smallest around the optimal point $\varepsilon_q' = \varepsilon_q \approx 0$.

and $\varepsilon_c^{(\mathrm{dip})}$ exist around the $\varepsilon_q' \sim 0$ region in Fig. 3. At $\varepsilon_q' \sim 0$, the qubit energy splitting is a quadratic function of the qubit bias. Thus, qubit state is insensitive to charge noises that lead to qubit dephasing, and this zero bias point corresponds to an optimal point [8].

The third terms of Eq. (5) decrease $\varepsilon_d^{(\mathrm{peak})}$ and $\varepsilon_c^{(\mathrm{dip})}$ when $\varepsilon_q' < 0$ and increases them when $\varepsilon_q' > 0$ ($\beta\Delta \gg 1$),

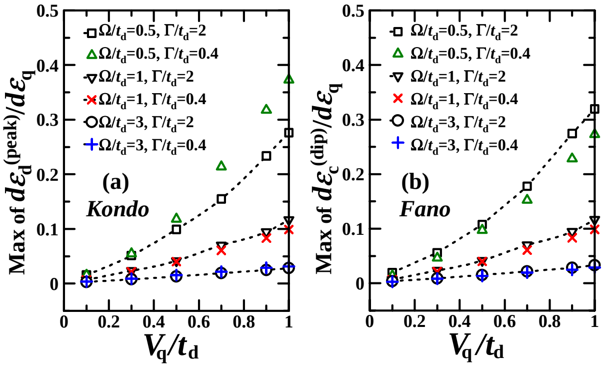

FIG. 4: Maximum values of $d\varepsilon_d^{(\mathrm{peak})}/d\varepsilon_q$ and $d\varepsilon_c^{(\mathrm{dip})}/d\varepsilon_q$ plotted as a function of the qubit-detector coupling V_q for both fast ($\Gamma/t_d = 2$) and slow ($\Gamma/t_d = 0.4$) detectors for (a) the Kondo and (b) the Fano-Kondo detectors. It can be seen that the maximum values do *not* vary with the speed Γ of the detectors. It can also be seen that $d\varepsilon_d^{(\mathrm{peak})}/d\varepsilon_q \propto V_q/\Omega$ and $d\varepsilon_c^{(\mathrm{dip})}/d\varepsilon_q \propto V_q/\Omega$.

resulting in the minimum structure of Fig. 3 at the optimal point $\varepsilon_q' = 0$. The derivatives, $d\varepsilon_d^{(\mathrm{peak})}/d\varepsilon_q$ and $d\varepsilon_c^{(\mathrm{dip})}/d\varepsilon_q$, have their maximum values around the middle points between the minimum of the optimal points and the small ε_q region as shown in Fig. 4. It can be seen that: (i) the maximum values of $d\varepsilon_d^{(\mathrm{peak})}/d\varepsilon_q$ and $d\varepsilon_c^{(\mathrm{dip})}/d\varepsilon_q$ do not depend on the speed Γ of the detectors, and (ii) there is a relationship between the peaks and V_q/Ω such as $\mathrm{Max}(d\varepsilon_d^{(\mathrm{peak})}/d\varepsilon_q) \propto V_q/\Omega$, and $\mathrm{Max}(d\varepsilon_c^{(\mathrm{dip})}/d\varepsilon_q) \propto V_q/\Omega$ when $\Omega/t_d > 1$. These weak dependences of the maximum values on the speed Γ of the detectors are considered to be because of the sharp response of the Kondo and Fano-Kondo effects at their coherent extrema. In principle, V_q can be calculated from the structure of the system. Thus, in experiments, if we can prepare several samples with the different distances between the detector and the qubit, we can estimate the tunneling coupling Ω for the charge qubit by using these relations.

4. Coclusions

We have studied the Kondo and the Fano-Kondo effects in QD system from viewpoint of the detectors of a capacitively coupled charge qubit. We found that, by analyzing the derivatives of the shifts of the peak and dip as the function of the qubit bias, we can infer the tunneling strength between the states $|0\rangle$ and $|1\rangle$ of the charge qubit.

[1] S. Sasaki *et al*, Nature (London) **405**, 764 (2000).
[2] A.W. Rushforth *et al.*,Phys. Rev. B **73**, 081305(R) (2006).
[3] B.H. Wu *et al.*, Phys. Rev. B **72**, 165313 (2005).
[4] D.M. Newns and N. Read, Adv. Phys. **36**, 799 (1987).
[5] T. Tanamoto and Y. Nishi, Phys. Rev. B **76**, 155319 (2007).
[6] T. Tanamoto and K. Muraoka, Appl. Phys. Lett. **96**, 022105 (2010).
[7] T. Aono and M. Eto, Phys. Rev. B **63**, 125327 (2001).
[8] D. Vion *et al.* Science **296**, 886 (2002).