

Dynamic creation of a topologically-ordered Hamiltonian using spin-pulse control in the Heisenberg model

T. Tanamoto¹, K. Ono², Y.X. Liu^{3,4}, and F. Nori^{4,5}

¹Corporate R & D center, Toshiba Corporation, Saiwai-ku, Kawasaki 212-8582, Japan

Phone: +81-44-549-2192, E-mail: tetsufumi.tanamoto@toshiba.co.jp

²Low temperature physics laboratory, RIKEN, Wako-shi, Saitama 351-0198, Japan

³Institute of Microelectronics, Tsinghua University, Beijing 100084, China

⁴Center for Emergent Matter Science, RIKEN, Saitama 351-0198, Japan

⁵Department of Physics, The University of Michigan, Ann Arbor, Michigan 48109-1040, USA

Abstract: Hamiltonian engineering is an important approach for quantum information processing, when appropriate materials do not exist in nature or are unstable. Here, we theoretically propose methods to dynamically realize the Kitaev spin Hamiltonian and Yao-Kivelson (YK) Hamiltonian from the Heisenberg spin Hamiltonian using a pulse-control technique. We show that if we carefully design two-dimensional pulses, we can obtain the desired Hamiltonians in only one step.

1. Introduction

Topological quantum computation (TQC) has attracted considerable interest due to its robustness to local perturbations. Anyons, which obey different statistics from bosons and fermions, are also of fundamental interest in physics. Kitaev [1] provided an important exactly-solvable model of a spin-1/2 system on a honeycomb lattice with potential links to topological quantum computation:

$$H_K = -J_x \sum_{x\text{-links}} X_j X_k - J_y \sum_{y\text{-links}} Y_j Y_k - J_z \sum_{z\text{-links}} Z_j Z_k, \quad (1)$$

where X_j , Y_j and Z_j are the Pauli spin operators and the interaction type (x , y , and z links) depends on the direction of the bond between the two sites (Fig. 1). This spin model has been stimulated research on the physics of anyon systems, including Majorana fermions. Yao and Kivelson extended the model to accommodate the chiral spin state [2]. However, it is not easy to find materials which have such anisotropic interactions.

Here, dynamical methods are provided to design the Kitaev Hamiltonian and the YK Hamiltonian from the Heisenberg Hamiltonian:

$$H_S = \sum_{i<j} [J_x X_i X_j + J_y Y_i Y_j + J_z Z_i Z_j], \quad (2)$$

by applying a customized pulse sequence to H_S , like NMR, using the Baker-Campbell-Hausdorff (BCH) formula [3].

2. Dynamical creation of the Kitaev spin Hamiltonian

The target Hamiltonian H_{tgt} is approximately obtained using the BCH formula by $H_{\text{tgt}} = H_S + H_R$, such as $e^{-itH_S} e^{-itH_R} \approx e^{-it(H_S+H_R)} - t^2 [H_S, H_R]/2 + \dots$, when $J_\alpha t \lesssim 1$. The terms higher than t are the unwanted ones. The transformed Hamiltonian H_R is produced by using rotations of the Pauli operators, $X_i \rightarrow -X_i$, $Y_i \rightarrow -Y_i$ and $Z_i \rightarrow -Z_i$.

Direct method.— The direct way to convert Eq. (2) to Eq. (1) requires six steps, as shown in Fig. 2. The first

process is to create the three Ising Hamiltonians, $H_x = \sum_{i,j} J_x X_i X_j$, $H_y = \sum_{i,j} J_y Y_i Y_j$, $H_z = \sum_{i,j} J_z Z_i Z_j$, from Eq. (2) as shown in Figs. 2(a,c,e). The generated Ising Hamiltonians are described by $H_{\text{step1}}^\alpha = \log[e^{itH_S} e^{itH_{r1}^\alpha}]/(it)$, where $H_{r1}^\alpha = P_1^{\alpha\dagger} H_S P_1^\alpha$ ($\alpha = x, y, z$) is a rotated Hamiltonian by applying a $\pi/2$ -pulse around the α -axes on the lattice sites of Figs. 2(a,c,e). The next process is to erase unnecessary Ising interactions, such as $H_{\text{step2}}^\alpha = \log[e^{itH_{\text{step1}}^\alpha} e^{itH_{r2}^\alpha}]/(it)$, where $H_{r2}^\alpha = P_2^{\alpha\dagger} H_{\text{step1}}^\alpha P_2^\alpha$ is obtained by applying $\pi/2$ -pulses depending on the links in Figs. 2(b,d,f). Thus, the Kitaev Hamiltonian is dynamically obtained by $H_K^{\text{dir}}(t) = \log[e^{itH_{\text{step2}}^x} e^{itH_{\text{step2}}^y} e^{itH_{\text{step2}}^z}]/(it)$. Note that parts of the Kitaev Hamiltonian do not commute, *i.e.*, $[\sum_{x\text{-links}} X_j X_k, \sum_{y\text{-links}} Y_j Y_k] \neq 0$. Therefore, the unwanted terms increase in the direct method. A better method is presented below.

Efficient method.— The Kitaev Hamiltonian H_K is produced more efficiently from H_S when we apply rotation pulses more compactly. Figure 3 shows the distributions of the rotation pulses P_{eff} by which the BCH formula is used only once, such that

$$2\tau H_K^{\text{eff}} = \tau(H_S + H_R^{\text{eff}}), \quad (3)$$

with $H_R^{\text{eff}} = P_{\text{eff}}^\dagger H_S P_{\text{eff}}$. The x -link of the colored honeycomb is produced by applying a rotation around the y -axis and that around the z -axis on both sides of the link. Similarly, the y (z)-link is similarly produced.

Refresh overhead.— If τ_{rot} denotes the time of a single-qubit rotation, it takes $2(2\tau_{\text{rot}} + \tau)$ and $12(2\tau_{\text{rot}} + \tau)$ to produce rotations $\exp(-i2\tau H_K^{\text{eff}})$, and $\exp(-i4\tau H_K^{\text{dir}})$, respectively. Similarly to the conventional dynamic random access memory (DRAM), here we define the *refresh overhead* as the effectiveness of the refresh of the quantum state:

$$r_{\text{oh}} \equiv \text{refresh overhead} = \frac{\text{time required for refresh}}{\text{refresh interval}}. \quad (4)$$

The refresh overhead of the efficient method presented above is $r_{\text{oh}}^{\text{eff}} = \frac{2\tau_{\text{rot}} + \tau}{\tau} \approx 2J_z \tau_{\text{rot}} + 1$, and that of the direct method shown previously is $r_{\text{oh}}^{\text{dir}} = 3(2J_z \tau_{\text{rot}} + 1)$, for $J_z \tau \lesssim 1$. Thus, the efficient method is three times efficient than the direct method. When $\tau_{\text{rot}} \approx 1$ ns, the efficient method requires a time of $4\tau_{\text{rot}} \approx 4$ ns and the direct method requires a time of 24 ns. Thus, when the dephasing rate [4] is $T_2 \sim 10$ ns, only the efficient method is applicable. When the dephasing rate [5] is $T_2 \sim 100$ ns, both methods can be applied. A short τ_{rot} is important for both methods.

3. Numerical calculation—Fidelity.

Let us numerically estimate the improvement of the efficient method by calculating a *time-dependent gate fidelity* defined by $F(t) = |\text{Tr}[\exp(itH_K)U_P(t)]|/2^N$, where $U_P(t)$ denotes the evolution operator of the pulsed system. The gate fidelity shows how well the transformed Hamiltonian evolves compared with H_K .

Figure 4 shows the numerical result for $N = 10$ qubits (two honeycomb lattices), where $J_x = J_y = 0.3J_z$ including the the spin-orbit interaction and the hyper-fine interaction. In various parameter regions, the overlap with the Kitaev Hamiltonian is excellent when using the pulse-controlled method.

4. Dynamical creation of the Yao-Kivelson Hamiltonian

The above-mentioned efficient method is extended and applied to the one step generation method of the Yao-

Kivelson method as shown in Fig.5.

5. Conclusions

In summary, we proposed methods on how to dynamically generate the Kitaev spin Hamiltonian and the Yao-Kivelson Hamiltonian from the Heisenberg spin Hamiltonian by using a dynamical approach. We clarified that a dynamic TQC is available by periodically reproducing the topological Hamiltonian.

References

- [1]A. Kitaev, Annals of Physics **321**, 2 (2006).
- [2]H. Yao and S.A. Kivelson, Phys. Rev. Lett. **99**, 247203 (2007).
- [3]T. Tanamoto, Phys. Rev. A. **88**, 062334 (2013).
- [4]J.R. Petta, *et al.* Science **309**, 2180 (2005).
- [5]B.M. Maune, *et al.*, Nature **481**, 344 (2012).

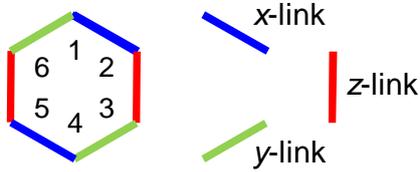


Fig.1:Kitaev model on a honeycomb lattice.

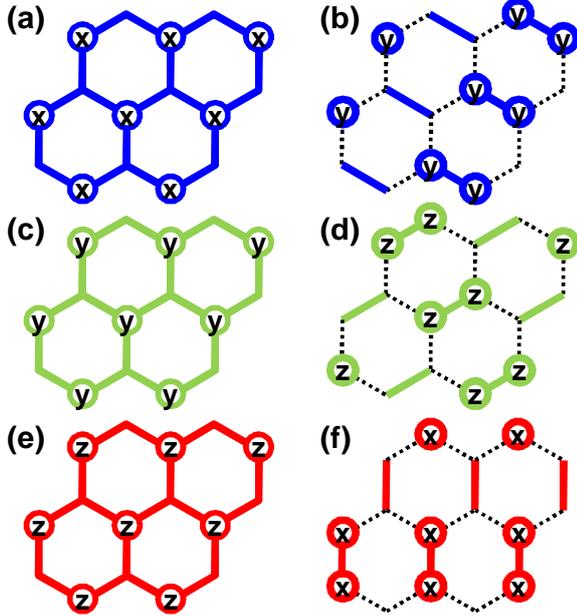


Fig.2:Direct method to dynamically produce a Kitaev Hamiltonian. The symbols x, y and z in the lattice sites show the application of $\pi/2$ -pulses around x, y and z, respectively. The bonds with dotted lines indicate that there is no interaction between the connected sites. (a) Pulse mapping of P_1^x to create the Ising Hamiltonian, $H_{\text{step}1}^x$.(b) Pulse mapping to select only the x-link of the Kitaev Hamiltonian from the Ising Hamiltonian of (a). (c)-(d) and (e)-(f) express pulse distributions for generating the y and z links, respectively

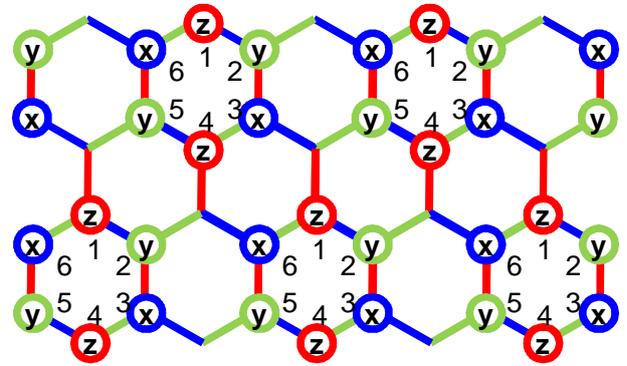


Fig.3: Efficient method to dynamically produce a Kitaev Hamiltonian from the Heisenberg model. The x, y and z on the lattice sites show the application of $\pi/2$ -pulses around x, y and z, respectively.

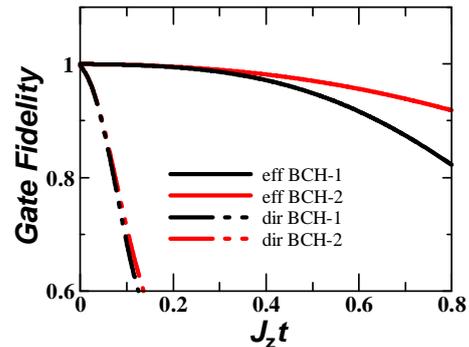


Fig.4: Numerically-calculated gate fidelity. Here "dir" corresponds to the direct method (Fig.2), and "eff" corresponds to the efficient method (Fig.3), respectively. "BCH-n" means that the BCH formula is applied n times.

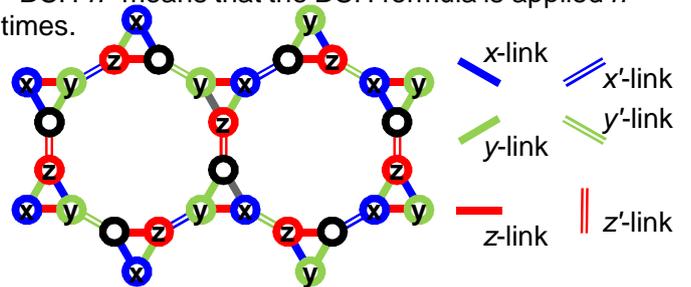


Fig.5: One step generation of the Yao-Kivelson model using pulse control.