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Topological Phenomena in Ultracold Atoms

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Abstract
Topological excitations depend on the symmetry of the order parameter manifold and exhibit material-independent universal features that are stable against local perturbations. Bose-Einstein condensates of ultracold atoms can accommodate a rich variety of topological excitations due to the spin degrees of freedom. In this talk, I will present an overview of some of the recent developments on this topic. The dynamical creation of topological defects following spontaneous symmetry breaking and non-Abelian quantum turbulence are also discussed.

1. Introduction

Bose-Einstein condensates (BECs) of ultracold atomic gases are very dilute with the atomic density five order of magnitude more dilute than that of the air and the temperature typically of the order of hundreds of nano-Kelvin. BECs are cooled by using lasers (laser cooling) and similar sets of optics can be utilized to manipulate and probe the system. Thus, one can literally see macroscopic quantum phenomena which are controlled with high precision. Naturally, atomic-gas BECs offer an ideal playground to investigate topological phenomena such as fractional vortices, skyrmions and knots [1]. These objects are topologically protected and stable against external perturbations because one has to make global rearrangements of the field configuration to destroy them and hence requires a substantial amount of energy. Topological excitations are therefore robust against compression, stretch and twist but not rotation of the system which is a global operation.

2. Spinor BECs

A BEC of spin-S atoms can be described by an order parameter having $2S$+1 components. Such a BEC with spin degrees of freedom is called a spinor BEC which has several unique properties arising from internal degrees of freedom and their coupling to the gauge degree of freedom [2][3]. The number of ground-state phases of spinor BECs at zero magnetic field is two, five and fourteen for S=1, 2 and 3 BECs [4]. Such a proliferation in the number of ground-state phases implies a rich variety of topological excitations. In fact, spinor BECs can accommodate almost all types of topological excitations [1].

Experimentally, spinor BECs are realized in ^{23}Na for S=1 [5], ^{87}Rb for S=1 [6] and 2 [7][8], and ^{52}Cr for S=3 [9]. The ground-state phase of a spin-1 ^{23}Na BEC is polar, whereas that of a spin-1 ^{87}Rb BEC is ferromagnetic. The ground-state of a spin-2 ^{87}Rb BEC is expected to be either uniaxial nematic or cyclic, where the cyclic phase has a tetrahedron symmetry and is known to possess non-Abelian vortices [10].

3. Topological Excitations

The order parameter of a spinless BEC is scalar and the topological aspect of the order parameter is represented by its phase factor. Let us consider a closed loop in real space and examine how much the phase changes as we make a complete circuit of the loop. The phase change must be an integer multiple of 2π due to the single-valuedness of the order parameter. This implies that a spinless BEC can accommodate U(1) vortices having integer winding numbers. The U(1) vortices were observed in a number of laboratories by mechanically rotating the system or by imposing a synthetic magnetic field [11].

The order parameter of a spin-1 ferromagnetic BEC is SO(3) which describes the direction of the magnetization and the rotation angle about it. The nontrivial topological excitation in this system is an SO(3) vortex, where the m =±1 components of the spin state have unit vortices with opposite signs and the m =0 component has no vorticity. Such a vortex has its core filled by the polar (i.e., m=0) component and therefore it is called a polar-core vortex. The SO(3) vortex has the $\mathbf{Z}_2$ symmetry and, as a consequence, two such vortices are combined to produce a uniform (trivial) texture. The polar-core vortex was observed in a spin-1 ^{87}Rb BEC [12].

The order parameter of a spin-1 polar BEC has, in an appropriate representation, only one nonvanishing component, i.e., the m=0 component. Let us consider a loop in real space and rotate the spin by π as we make a complete circuit around the loop. Then we can show that the spin part of the order parameter changes its sign. The single-valuedness of the order parameter dictates that the global (gauge) phase of the order parameter must change by π which is one half of the usual phase change of 2π for the U(1) vortex. Therefore the polar BEC can accommodate a half-quantum vortex [13] whose mass circulation is quantized in units of $h/(2M)$ rather than the usual h/M, where h is the Planck constant and M is the mass of the atom. A skyrmion texture was created in the polar phase of a ^{23}Na BEC [14]. However, the created skyrmion turned out to be dynamically unstable and it was suggested that in the decay process of the skyrmion a pair of half-quantum vortices were created.

The polar phase of a spin-1 BEC is also known to accommodate a knot excitation [15]. While vortices are characterized by the winding number, knots are characterized by the Hopf charge or the linking number whose physical meaning can be explained as follows. Suppose that we take a loop along which the local spin orients in the positive z direction. We also take another loop along which the local spin orients in a different direction, say the positive y direction. If the two loops link once, the linking number is one. Surprisingly, such exotic topological excitations have recently been realized in a spin-1 ^{87}Rb condensate by preparing it in the polar state [16]. Although the ground state of the spin-1 ^{87}Rb BEC is ferromagnetic, one can prepare it in a polar state by transferring all the atoms into the m=0 state. Such a state is not only an excited state but also dynamically unstable. However, the lifetime of the state is long enough to create a knot within the lifetime.

4. Non-Abelian Vortices

The ground-state phase of the spin-2 ^{87}Rb BEC lies close to the phase boundary between the uniaxial nematic phase and the cyclic phase [17][18]. The cyclic phase is known to host one-third and two-thirds vortices [19] due to the tetrahedral symmetry of the order parameter and non-Abelian vortices [10] because the generators of different types of vortices do not commute in general.

A distinctive feature of non-Abelian vortices manifests itself in the collision dynamics. When two Abelian vortices collide, they usually reconnect and create two new vortices. However, if they are non-Abelian, they do not reconnect but form a rung vortex that connects the two vortices. Here the rung vortex arises when the generators of two colliding vortices do not commute; then the nonvanishing commutator creates the rung vortex in the course of time evolution. Thus, every time non-Abelian vortices collide, they will be connected together by a rung vortex and the vortices will end up forming a large scale network of vortices. This fact has an important implication of quantum turbulence where the statistical laws of turbulence such as the Kolmogorov power laws are expected to be modified by the non-Abelian nature of vortices [20].

5. Conclusion and future prospect

When a topological invariant emerges in condensed matter physics, it could lead to a quantized value of a physical quantity such as the quantized Hall conductance and have an important implication in metrology. Ultracold atomic gases, on the other hand, have so far played a vital role in testing various theoretical predictions about topological quantum phenomena as discussed above. In fact, ultracold atomic gases have been instrumental in investigation of the dynamics of spontaneous symmetry breaking such as the Kibble-Zurek mechanism [21][22]. On the other hand, quantum gases have also been demonstrated to play a pivotal role in precision measurements such as optical lattice clocks [23] and the test of equivalence principle [24]. One cannot help but wonder whether and, if so, how quantum gases and topological quantum phenomena can conspire to yield as yet envisaged precision metrology and topological quantum computation.

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