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# Relaxation Oscillations in a Nonlinearly Driven GaAs MEMS Resonator

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**Abstract**

**In this work we investigate the behavior of a piezoelectric MEMS resonator, driven nonlinearly by an external source with two slightly differing frequencies. Depending on the detuning of the drive tones with respect to the MEMS resonance frequency, we observe a qualitative change in behavior that is due to the creation of a relaxation oscillation cycle, which can be reproduced using a simplified model. Such oscillations may be used as a Pulse Width Modulation (PWM) readout mechanism for nonlinear MEMS sensors.**

## 1. Introduction

MEMS devices are ubiquitously present around us in the form of acceleration, rotation, pressure, and force sensors. In addition to their role as sensors and actuators, MEMS systems offer an excellent platform for the exploration of nonlinear dynamical effects. Indeed, it has been shown that nonlinear MEMS resonators offer the potential of improved sensitivity and new functionalities if operated properly [1]-[2].

It is of continuing interest to explore the rich terrain of nonlinear dynamics in MEMS devices in the aim of adapting it for use in engineering applications. For example, nonlinearly driven MEMS cantilevers were shown to exhibit a higher sensitivity as gas sensors compared to their linear counterparts [3]. Nevertheless, this increased sensitivity is usually associated with more complex transduction and detection techniques. It is therefore of interest to find an approach that combines both the enhanced performance of nonlinearly operated MEMS devices, and a simplified detection that is compatible with ultra-low power sensor nodes applications.

In this work we look at the response of a MEMS resonator driven into the nonlinear regime, we show that upon driving such a resonator with two-tones of slightly different frequencies, we observe a qualitative change in behavior that strongly depends on the excitation frequency. The behavior is best explained by the creation of a relaxation limit cycle [4] which results in a pulse-like output with sudden transitions.

## 2. Experimental Setup and Results

The device used in this work is a clamped-clamped beam MEMS structure that is 150 μm long, 20 μm wide and fabricated from an GaAs/n-AlGaAs modulation-doped heterostructure of 100 nm thickness each. Near the clamping on each end of the beam a gold top electrode is deposited, while a contact to the underlying conductive Two-diemnsional electron gas (2DEG) layer is established using gold-germanium-nickel alloy. The heterostructure acts as a capacitive piezoelectric transducer upon the application of an excitation voltage. The MEMS device is mounted inside a vacuum chamber (< $10^{-4}$ Pa) and excited electrically using an WF1974 signal generator, while its motion is probed using a NEOARK Laser Doppler Vibrometer (LDV).

When the device is excited with a weak actuation signal (~ 5 mV), we obtain a linear second harmonic oscillator response with a resonance frequency of 1.57 MHz and a quality factor of 628 corresponding to a linewidth of 2.5 kHz. As the drive amplitude is increased the response of the resonator exhibits a frequency shift characteristic of a Duffing oscillator with a positive nonlinear coefficient.

We drive the resonator with two frequencies of equal amplitude and detuned by 10 Hz ($\omega_d$, and $\omega_d + 20\pi$ Hz). We observe the effect of frequency mixing caused by the nonlinearity, as shown in Fig. 1 (red trace) for $\omega_d$ = 1.5778 MHz. As we change the drive tone frequency slightly to $\omega_d$ = 1.5779 MHz, we observe a sudden change in the measured spectrum (blue trace).

In the time domain, when the system is observed at time scales corresponding to ~ 10 Hz using an HP89410A Vector Signal Analyzer (VSA), the difference is equally pronounced. With the system showing a pulse like response with sharp transitions in one case and smoothly varying one in another as shown in Fig. 2.

## 2. Theory and Modeling

To explain our observation we qualitatively model the system using the Duffing nonlinear oscillator equation [6]:

$$\ddot{X} + \gamma\dot{X} + \omega_0^2 X + \alpha X^3 = F_1 cos(\omega_1 t) + F_2 cos\big((\omega_1 + \delta)t\big) \quad (1)$$

where $X$ signifies the oscillator's motion, $\gamma$ the damping, $\omega_0$ its natural frequency, $F_1$ and $F_2$ the forcing terms, and $\delta$ the frequency detuning between the two tones (with $\delta << \gamma$). Note the presence of two driving force terms on the right hand side of eq. (1). We focus on the interesting case of $F_1 = F_2 = F$.

For a single driving force, the response of an oscillator is well known to exhibit bistability, a region where two stable solutions coexist. An example simulation is shown in Fig. 3, with the bistability region shown as light-green area.

By adding equal force terms, and solving using the rotating frame approximation, eq. (1) reduces to:

$$(9/16)\alpha^2 X^6 - 3\alpha\omega_0^2\Delta X^4 + (4\Delta^2 + \gamma^2)\omega_0^2 X^2 = 2F(1 + \cos(\delta t)) \quad (2)$$

where $\Delta = \omega_I - \omega_0$, and $\omega_I$ is the drive frequency.

Thus the addition of another forcing term, which is slightly detuned from the first drive tone, results in a force envelope that slowly oscillates between 0 and $2F$. Depending on $\Delta$, the slowly oscillating drive may either force the system to jump from one solution branch to the other, thus establishing a relaxation cycle with sharp transitions as seen experimentally (red trace in Fig. 2.b), or alternatively to remain on a single branch thus exhibiting a slowly varying amplitude.

## 3. Conclusions

This work demonstrates that by driving a nonlinear oscillator with two adjacent frequency tones, a relaxation oscillation cycle maybe induced. The relaxation oscillations appear as pulses in the system's response, the existence and width of these pulses vary depending on the amplitude and position of the driving tones. Thus the output from a nonlinear oscillator under dual tone drive can be used as a simple readout mechanism for MEMS sensors.


## References

[1] Y. Hiroshi, *Semiconductor Science and Technology* **32** (2017) 103003.

[2] D. S. Greywall, et al., *Physical Review Letters*, **72** (1994) 2992.

[3] V. Warner, et al., *Nanotechnology* **25** (2014) 425501.

[4] S. Houri, et al., *Applied Physics Letters* **107** (2015) 073502.

[5] E. A. Krömmer, et al., *Applied Physics Letters*, **77** (2000) 3102.

[6] I. Kovacic, and M. J. Brennan. The Duffing equation: nonlinear oscillators and their behavior, *John Wiley & Sons*, 2011.


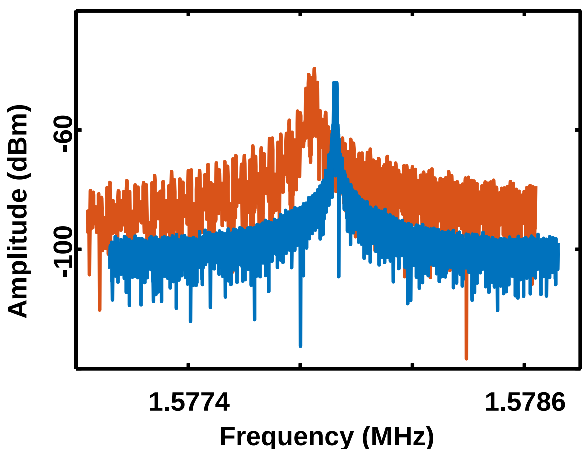


Fig. 1 Frequency domain response to two-tone driving, showing the large effect that slight detuning has, for $\omega_d$ = 1.5778 MHz (red), and $\omega_d$ = 1.5779 MHz (blue). $V_{drive}$ = 0.75 V.

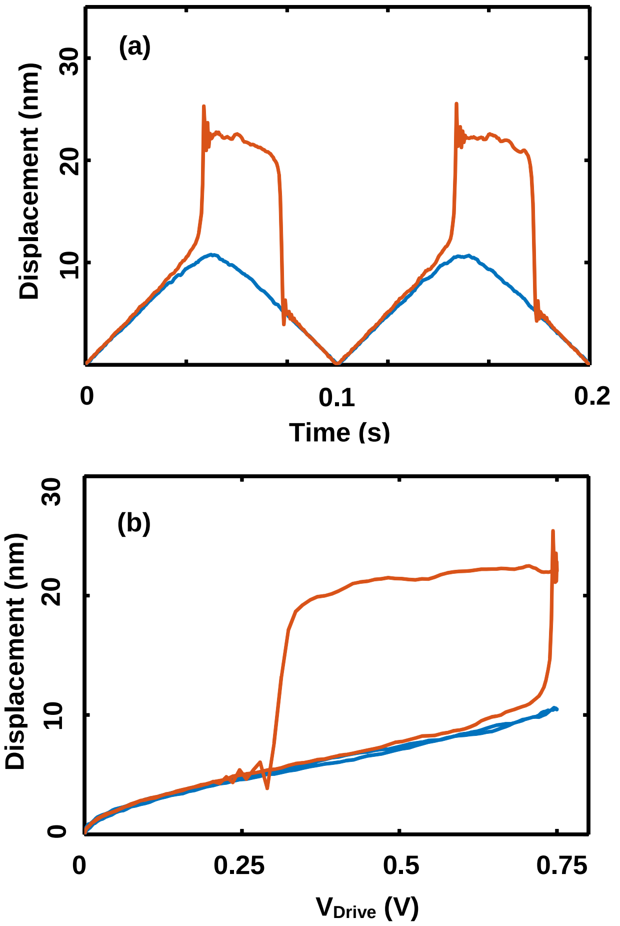


Fig. 2 Experimentally obtained time domain trace (a), showing a pulse-like or a smoothly oscillating output (red and blue traces respectively), depending on whether or not a relaxation limit cycle exists. (b) The voltage-displacement plot of the oscillator.

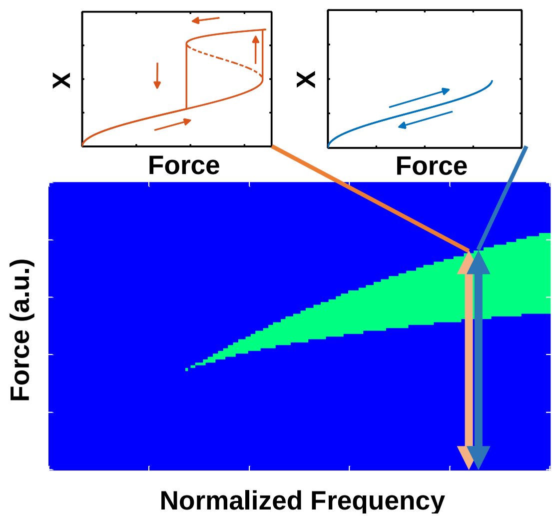


Fig. 3 Simulation showing bistability (green) in a nonlinear oscillator. If $2F$ exceeds the bistability region the oscillator undergoes a relaxation cycle (red plot), a slight detuning keeps $2F$ within the bistability area, only slow oscillations are observed (blue plot), similar to Fig. 2b.