# Non-Markovian effect on spin pumping

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## Abstract

Using a minimum model consisting of a magnetic quantum dot and an electron reservoir, we investigate spin pumping by its precessing magnetization. When the time scale of the precession period is as short as the correlation time of the reservoir, non-Markovian effect on the electron dynamics underlying the spin pumping should be considered. In this study, we analyze the electron dynamics by means of the full counting statistics embedded in the quantum master equation formalism, and obtain frequency dependence of the spin pumping both in Markovian case and non-Markovian case. By comparing the results, we show that the Markovian approximation causes an overestimation of the pumped quantity especially in a high frequency regime.

### 1. Introduction

The generation of a spin-polarized electron current (spin current) in a microscopic system such as quantum dot is a central issue in realizing single electron spintronics. The spin pumping is a method to generate spin current in a ferromagnet/normal metal junction, where the spin current is induced by precession of the magnetization in the ferromagnet into the normal metal [1]. The method has been widely used in experiments in bulk systems [2]. A microscopic model used in analyzing the spin pumping consists of a magnetic quantum dot attached to an electron reservoir and is called a minimum model [3]. In the minimum model, the spin current is induced into the electron reservoir by time-periodic precession of the magnetization of the dot.

In previous studies, the effect has been formulated in an adiabatic regime, where the precession is sufficiently slow compared with the relaxation time of the dot by tunneling. However, since period of the precession ( $\approx 10^{-10}$  sec.) is comparable to or shorter than the relaxation time (  $\approx$  $10^{-9}$  sec.) in an actual experiment [4], it is necessary to study the spin pumping under relaxation. In light of the perspective, the authors studied it by applying the full counting statistics embedded in the quantum master equation formalism [5] with the Born-Markov approximation, which is verified when the time scale under consideration is sufficiently longer than the correlation time of the reservoir [6]. However, in the short time scale studied in the experiment, the correlation time of the reservoir should also be considered to be finite. This requires to analyze electron dynamics beyond the Markovian approximation to obtain a precise description of the spin pumping especially in a high frequency regime.

In this study, we study the non-Markovian effect on the spin pumping by applying the full counting statistics embedded in a non-Markovian quantum master equation formalism to the minimum model. By including the non-Markovian effect, the electron dynamics exhibits a time-reversible behavior in the early stage of the relaxation process within the short time scale. We discuss effects of the partial time-reversibility on the spin pumping.

### 2. Formalism

We consider the minimum model of the spin pumping [3] consisting of a ferromagnetic quantum dot contacting with an electron lead (FIG.1). The quantum dot has a dynamic magnetization M(t) that rotates around a fixed axis with a certain period T. An electron in the dot is spin polarized because of the s-d exchange interaction with the magnetization. The lead is modeled by an ensemble of free electrons. The coupling between the dot and the lead is assumed to be spin preserving and is modeled by Ohmic spectral density.

We formulate the spin pumping by using the full counting statistics. It provides statistics of net number of transferred electrons from the dot to the lead within a certain time interval by using difference of outcomes of two successive projective



FIG. 1 A schematic figure of the minimum model. Precession of the magnetization M(t) in the quantum dot may induce spin current into the electron reservoir (lead).

measurements of the electron number in the lead. By obtaining mean values of transferred spin up and spin down electrons  $\langle \Delta n_{\uparrow} \rangle$  and  $\langle \Delta n_{\downarrow} \rangle$ , we define the spin current by  $I_{\uparrow} = (\langle \Delta n_{\uparrow} \rangle - \langle \Delta n_{\downarrow} \rangle)/T$ . The mean values can be evaluated by solving a time-convolutionless type quantum master equation modified to include information of the counting [7]. In the present study, we evaluate the spin current by numerically solving the master equation with and without the Markovian approximation.

#### 3. Results

In FIG. 2, we plot the frequency dependence of the spin pumping obtained by numerically solving the master equation with (blue line) and without (red line) the Markovian approximation. The insets show a magnification of a low frequency regime (up to  $\overline{\Omega} = 0.005$ ) (i) and dependence in a higher frequency (up to  $\overline{\Omega} = 0.05$ ) (ii).

In both cases, the spin current linearly depends on  $\Omega$  for a lower frequency, whereas it oscillates depending on  $\Omega$  for a higher frequency. The oscillation is caused by a Rabi oscillation between the spin up state and the spin down state in the dot, where the spin current takes a maximum value when the precession period is an integer multiple of the Rabi period and it takes a minimum value when the precession period is a half integer multiple of the Rabi period [5].

By comparing the Markovian and the non-Markovian results, we find that both results coincide in the linear regime, whereas the Markovian result deviates from the non-Markovian result in the oscillating regime. The deviation is significant in the higher frequency region presented in the inset (ii). The deviation is caused by a backflow of electrons from the lead to the dot, which is a temporal reversal of the electron flow due to the partial time-reversibility of the non-Markovian dynamics [8]. The result shows that the backflow reduces the amount of the spin current. Conversely, since the non-Markov dynamics is more realistic than the Markovian dynamics especially in the high frequency region, one may overestimate the spin current relying on the Markovian approximation.



FIG. 2 A typical frequency dependence of the spin pumping evaluated with (blue line) and without (red line) the Markovian approximation. The axis are normalized by a unit angular frequency  $\omega_u$ , which is the angular frequency of the Rabi oscillation between spin up and spin down states in the dot.

# 3. Conclusions

In this study, we have studied the spin pumping in the minimum model by analyzing the electron dynamics in terms of the full counting statistics embedded in the quantum master equation formalism with and without the Markovian approximation. By comparing the Markovian result and the non-Markovian result, we find that the backflow of electrons caused by the non-Markovian dynamics reduces amount of pumped electron current. Since the non-Markovian dynamics is more realistic than the Markovian dynamics especially in the high frequency region, one may overestimate the spin current relying on the Markovian approximation. Thus the non-Markovian dynamics should be taken into account when one estimate the pumping under the rapid modulation.

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