A Dependence of the Skyrmion Hall Effect on the Gilbert Damping Constant

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Abstract

We study the skyrmion Hall effect in the heterostructures of heavy metal/ultrathin ferromagnet/insulator multilayers by micromagnetic simulation. Under a small Gilbert damping constant, it is found that there exists a meaningful difference between the result of Hall angle using micromagnetic simulation and that of the Thiele equation. Therefore, we conclude that the Thiele equation is not valid when the Gilbert damping constant is small.

1. Introduction

Magnetic skyrmions are chiral spin structures with particlelike nature. They are relatively stable against perturbations, which originates from their topologically protected field configurations. Therefore, it is difficult to destroy their structures and deform them to other magnetic structures. Recently, magnetic skyrmions have been paid much attention due to potential ability for high-density memories and logic devices in the field of spintronics. This is because magnetic skyrmions are stable and can be driven by a very low current density in comparison with the critical current density for domain wall motion.

In heterostructures of heavy metal/ultrathin ferromagnet/insulator multilayers, magnetic skyrmions can be moved by the spin orbit torque from the spin Hall effect of the heavy metal layer. In this situation, magnetic skyrmions move along the perpendicular direction to the current flow. This effect is called as the skyrmion Hall effect [1]. In this letter, we study the skyrmion Hall effect using micromagnetic simulation. As a result, we show its dependence on the Gilbert damping constant and show that the Thiele equation is invalid for the system in the case of $\alpha \ll 1$.

2. Theory

The dynamics of the magnetization under any spin torque is governed by the modified Landau-Lifshitz-Gilbert (LLG) equation [2, 3]:

$$\frac{dM}{dt} = -\gamma \mu_0 M \times H_{\text{eff}} + \frac{\alpha}{M_s} M \times \frac{dM}{dt} + \tau, \qquad (1)$$

where M is the magnetization vector, M_s is the saturation magnetization, γ is the gyromagnetic ratio, μ_0 is the vacuum permeability, α is the Gilbert damping constant, H_{eff} is the effective magnetic field, and τ is any torque working on the magnetization. The effective magnetic field represents all the effects working on magnetic moments and is written in terms of functional derivative as follows:

$$H_{\rm eff} = -\frac{1}{\mu_0} \frac{\delta E[M]}{\delta M},\tag{2}$$

where *E* is the total energy of all the effects working on magnetic moments. In this study, the total energy consists of the exchange energy, uniaxial anisotropic energy, interfacial Dzyaloshinskii-Moriya interaction (DMI) energy, and the demagnetization energy. For the demagnetization energy, we assume that the demagnetization field H_d is determined by $H_d = -M_s m_z \hat{z}$. Moreover, we utilize the spin orbit torque from the spin Hall effect to move the magnetic skyrmion, which is given as follows [4]:

$$\tau = \frac{\gamma \hbar}{2e} \frac{\theta_{\rm sh}}{t_f M_s} m \times [m \times (\hat{z} \times \boldsymbol{j})], \qquad (3)$$

where \hbar is the Dirac constant, *e* is the electron charge, *m* is the unit magnetization vector, θ_{sh} is the spin Hall angle of the heavy metal, t_f is the thickness of the ferromagnet layer, and *j* is the current density.

Steady state motion of magnetic structures is represented by the Thiele equation [5] which is obtained from the LLG equation (1) and is written for a magnetic skyrmion under the influence of the spin Hall torque as follows [1]:

$$\boldsymbol{G} \times \boldsymbol{v} - 4\pi \alpha \boldsymbol{\mathcal{D}} \cdot \boldsymbol{v} + 4\pi \boldsymbol{\mathcal{B}} \cdot \boldsymbol{j} = \boldsymbol{0}, \tag{4}$$

where $G = (0, 0, -4\pi Q)$ is the gyromagnetic coupling vector, $Q = \frac{1}{4\pi} \int \boldsymbol{m} \cdot \left(\frac{\partial \boldsymbol{m}}{\partial x} \times \frac{\partial \boldsymbol{m}}{\partial y}\right) dx dy$ is the topological charge of the magnetic skyrmion, \boldsymbol{v} is the magnetic skyrmion velocity, $\mathcal{D} = \begin{pmatrix} \mathcal{D} & 0\\ 0 & \mathcal{D} \end{pmatrix}$ is the dissipative force tensor, $\mathcal{B} = \begin{pmatrix} \mathcal{B} & 0\\ 0 & \mathcal{B} \end{pmatrix}$ shows the efficiency of the spin Hall torque, and the components of these tensor \mathcal{D} and \mathcal{B} are determined by the configuration of the magnetic skyrmion. When the current density is spatially homogeneous and has only *x*-component $\boldsymbol{j} = (\boldsymbol{j}_x, 0)$, the magnetic skyrmion velocity is given by

$$v_x = \frac{\alpha \mathcal{D}}{Q^2 + \alpha^2 \mathcal{D}^2} \mathcal{B} j_x, \quad v_y = -\frac{Q}{Q^2 + \alpha^2 \mathcal{D}^2} \mathcal{B} j_x.$$
(5)

Therefore, the ratio of in-plane velocity components is given by

$$\frac{v_y}{v_x} = \frac{-Q}{\alpha \mathcal{D}}.$$
 (6)

3. Results and discussion

In order to solve the LLG equation numerically, we use the parameters $M_s = 600 \text{ kA/m}$, the exchange stiffness A = 30 pJ/m, the DMI constant $D = 4 \text{ mJ/m}^2$, and the anisotropy constant $K = 0.8 \text{ MJ/m}^3$, $t_f = 1 \text{ nm}$, $\theta_{\text{sh}} = -0.1$. For the initial state of the LLG simulation, we set a magnetic skyrmion which is obtained by the variational principle [6]. Then, we apply the current density $j_x = 1 \text{ TA/m}^2$ and the magnetic field $B_z = 1 \text{ mT}$. As shown in Fig.1, a magnetic skyrmion moves to the y direction which is perpendicular to the direction of the current.



Fig. 1 The skyrmion Hall effect in the case of $\alpha = 0.3$. The blue dotted line is a guide for eyes.

From the numerical results, we calculate the ratio of components of in-plane average velocity and compare it with the analytical value which can be obtained from the eq. (6) as shown in Fig.2. In order to obtain the analytical value, we need to calculate \mathcal{D} . Wherein, we have used the magnetization distribution of the skyrmion obtained by the variational principle. As shown in Fig. 2, it is found that there exists large difference between the result of the micromagnetic simulation and that of the Thiele equation under the condition of $\alpha \ll 1$.



Fig. 2 The dependence of the ratio of in-plane velocity components on the damping constant α . The blue solid circles and the orange solid line represent the numerical results and analytical value obtained from eq. (6), respectively.

It can be considered that the distortion of the structure of the magnetic skyrmion becomes larger with decreasing the value of α since the direction of the magnetization inclines more slowly to the direction of the effective magnetic field with decreasing the value of α as shown in Fig. 3. Namely, the systems under very small α do not satisfy the assumption of the Thiele equation that the motion of magnetization structure is only translation and its configuration does not change.



Fig. 3 The illustration of the magnetization damping. Here, the red and blue solid arrows are the magnetization unit vector m and the effective magnetic field unit vector h_{eff} , respectively. The green and the orange solid curves represent the trajectories of the magnetization for the large and small α , respectively.

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