

## Environmental engineering for quantum energy transport

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### Abstract

**We present a way to accelerate quantum energy transport with environmental engineering. More concretely, we consider to apply stochastic noise with spatio-temporal correlation on a linear chain model. We show an acceleration of energy transport by extending the spatial correlation into the negative region (anti-correlation). We also show non-monotonic dependence on both the population time evolution and the transfer efficiency measures, which means that the correlation time needs to be chosen to achieve the optimum energy transport. These results show new possibilities to understand efficient energy transport in nature and engineer it to our technologies.**

### 1. Introduction

To transport quantum particles, noise affected by its environment has been widely considered as an obstacle to be eliminated. However, studies on energy transfer in green sulfur bacteria has opened a new way of thinking [1]: The bacteria generate energy to live by photosynthetic reaction. But, since the bacteria live in the deep bottom of lakes where only very dim light is present, it is necessary to transfer even an excitation by a single photon very efficiently from the light harvesting antenna complex to the photosynthetic reaction center. In the light harvesting antenna, called Fenna-Mathews-Olson(FMO) complex, an electron excitation generated by light is carried to the reaction center through bacteriochlorophyll (BChl) molecules. Since the molecules are surrounded by the protein molecules, thermal fluctuation of the protein molecules seems to disturb the energy transfer as noise. But recently it is found that environmental noise can assist the energy transfer on the assumption that the white noise is applied to the excited sites, which means that the environmental effect is treated in the Markovian approximation [2]. The results from Rebentrost [2] mean that excitation transfer can be controlled with a stochastic noise. In our work, we extend this study to treat colored noise with a finite correlation time and find optimum conditions for efficient quantum transport surpassing the white noise situation.

### 2. Results

#### Hamiltonian

The results by Rebentrost [2] mean that excitation transfer can be controlled with a stochastic noise. In our work, we extend this study to treat colored noise with a finite correlation time and find optimum conditions for efficient quantum transport surpassing the white noise situation. Our

model (schematic depicted in Figure 1) can be represented by the Hamiltonian

$$\mathcal{H}(t) = \mathcal{H}_0 + \epsilon \mathcal{H}_1(t) \quad (1)$$

with

$$\mathcal{H}_0 = \hbar \sum_{n=1}^N \omega_n |n\rangle \langle n| + \hbar \sum_{n < m} V_{nm} (|m\rangle \langle n| + |n\rangle \langle m|) \quad (2)$$

$$\mathcal{H}_1(t) = \hbar \sum_{n=1}^N f_n(t) |n\rangle \langle n| \quad (3)$$

where  $|n\rangle$  is the  $n$ -th excitation cite,  $\omega_n$  is the Larmor frequency of the  $n$ -th cite,  $V_{nm}$  is the transition frequency between the  $n$ -th and  $m$ -th cite, and  $f_n(t)$  is the fluctuating frequency on the  $n$ -th cite with a finite correlation time (Fig.1).

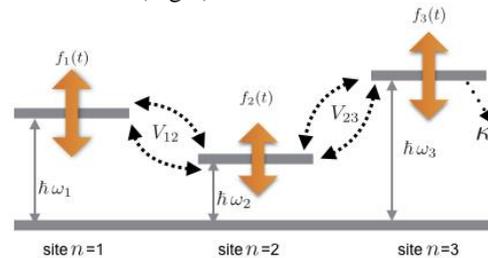


FIG. 1. A schematic picture of the multi-site model. This figure shows the case for the three-site model. The final site in the chain, i.e. site  $n=3$  in this illustration, includes a decay channel with rate  $\kappa$  to trap the system in its ground state. This corresponds to the excitation leaving the chain.

As the fluctuation, we assume that the average for each site is zero as  $\langle f_n(t) \rangle = 0$  and we describe the correlation function of these fluctuations  $\langle f_n(0) f_m(t) \rangle$  with a simple exponential decay as

$$\langle f_n(0) f_m(t) \rangle = c_{n,m} \Delta_{n,m}^2 \exp[-|t|/\tau_{c,\{n,m\}}], \quad (4)$$

where we introduce the quantity  $c_{n,m}$  for  $n, m = \{1, 2, \dots, N\}$  to allow for both positive and negative (anti) spatial correlations. It is defined over the range by  $-1 \leq c_{n,m} \leq 1$  where the extremal values  $-1$  ( $+1$ ) correspond to perfectly anti-correlated (correlated) noise respectively.  $\Delta_{n,m}$  is the amplitude of the fluctuation, and  $\tau_{c,\{n,m\}}$  is the correlation time of the fluctuation.

#### Formulation

To describe the dynamics of this model, we used the time-convolutionless type of master equation written as

$$\frac{d}{dt} \rho(t) = \langle (-i\mathcal{L}_0) \rangle \rho(t) + \int_0^t \langle (-i\hat{\mathcal{L}}_1(0)) (-i\hat{\mathcal{L}}_1(-\tau)) \rangle - \langle (-i\hat{\mathcal{L}}_1(0)) \rangle \langle (-i\hat{\mathcal{L}}_1(-\tau)) \rangle d\tau \rho(t)$$

In the master equation, we defined as  $\mathcal{L}_0 X \equiv \frac{1}{\hbar}[\mathcal{H}_0, X]$  and  $\mathcal{L}_1(t) X \equiv \frac{1}{\hbar}[\mathcal{H}_1(t), X]$  for an arbitrary operator  $X$ .

To describe the efficiency of the transport, we used average trapping time[3]

$$\langle t \rangle = \sum_n \int_0^\infty dt \rho_{nn}(t),$$

where  $\rho_{nn}(t)$  is the population of the excited state at the  $n$ -th site. The average trapping time indicates how long the population of the initial excitation remains in the system. Let us show you the results for 2-site and 3-site model in the following.

### 2-site model

Setting the initial condition as the full excitation at the site 1, we obtained the time evolution of the probability of finding the second site in the excited state. In Fig. (2a), we show the case where the fluctuation has no spatial correlation between sites. Fig. (2b) represents the situation where the fluctuation is anti-correlated between sites 1 and 2 by setting  $c_{1,2} = c_{2,1} = -1$ . In both figures, the insets show the short-time behavior.

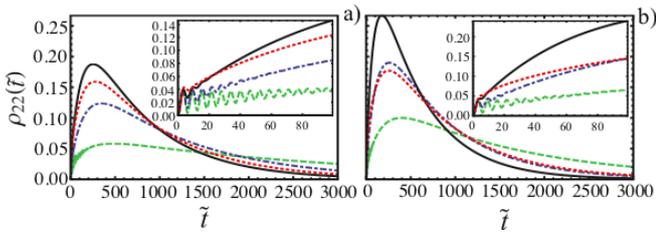


Fig.2 Time evolution of  $\rho_{22}(\bar{t} = \Delta t)$  between energy fluctuation is shown in (a) with spatially-uncorrelated noise, and in (b) with spatially-anti-correlated noise. Each plots corresponds to a different correlation time,  $\alpha = \Delta \cdot \tau_c$  as 0.1,0.3,1 and 10: the dashed lines corresponds to  $\alpha = 0.1$ , dotdashed lines to  $\alpha = 0.3$ , solid lines to  $\alpha = 1$  and dotted lines to  $\alpha = 10$ . The system parameters are set as,  $\epsilon^2 = 0.1$ ,  $\omega_1/\Delta = 1.5$ ,  $\omega_2/\Delta = 0.5$ ,  $V_{12}/\Delta = 0.1$ ,  $\kappa/\Delta = 0.005$ .

In Fig.2, we find that the spatially-anti-correlated noise shows the acceleration of energy transport compared with the spatially-uncorrelated case.

Next, we show the dependence of the average trapping time on degree of spatial correlation:  $c_{1,2} = c_{2,1} = c$ . Since the excitation trapping only at the second  $\mathcal{K}$  site with rate leads to  $\int_0^\infty dt \rho_{22}(t) = \kappa^{-1}$  we focus on the average trapping time  $\langle t \rangle$  minus an offset as  $\kappa^{-1}$ , which is drawn in Fig. 3.

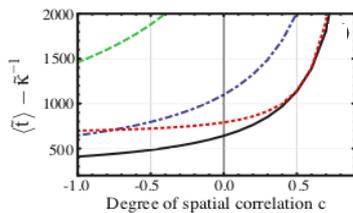


Fig.3 Two-site transport properties and their dependence on the degree of spatial correlation.

### 3-site model

We extend the above model to three-site linear chain under the nearest neighbor interaction with the interaction strengths given by  $V_{12} = V_{21} = V_{23} = V_{32} = V$  and  $V_{13} = V_{31} = 0$ . In Fig.4, we show the average trapping time  $\langle t \rangle - \kappa^{-1}$  for three-site model depending on the degree of spatial correlation for  $c_{2,3}(=c_{3,2})$ ,  $c_{1,2}(=c_{2,1})$ ,  $\alpha = 0.3$  with and while setting  $c_{1,3} = c_{3,1} = c_{2,3}c_{1,2}$ .

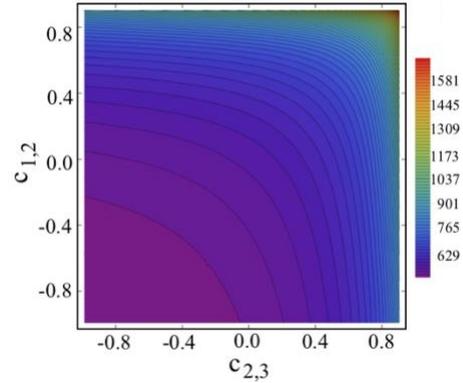


Fig.4 Average trapping time  $\langle t \rangle - \kappa^{-1}$  for three-site model.

In Fig.4, we find that the anti-correlated between the nearest neighbor sites shows the most efficient transport.

### 3. Conclusions

In this work we have considered the transport of excitation for a multi-site linear chain model whose energy levels are affected by spatio-temporal correlated stochastic noise processes. We find that the energy transport can be accelerated by extending the spatial correlation into the negative region (anti-correlation). These results show new possibilities to understand efficient energy transport in nature and engineer it to our technologies.

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