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Quantum annealing with capacitive-shunted flux qubits

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Abstract

Quantum annealing (QA) is a way to tackle combinatorial optimization problems by using Ising type interaction. Most of the previous demonstration of the QA has been done with superconducting flux qubits (FQ). The FQs used in these demonstrations has a short coherence time such as tens of nanoseconds. To exploit the quantum advantage, it is beneficial to use qubits with better coherence time. Here, we propose the QA with capacitive-shunted flux qubits (CSFQs) that has a few orders of magnitude better coherence time than that of the FQ used in the QA. While it is difficult to perform the conventional QA with the CSFQs due to the form and strength of the interaction between them, we theoretically show that the spin-lock based QA can be implemented by using the CSFQs even with the current technology. Our numerical results also support the feasibility of our proposal.

1. Introduction

Quantum annealing (QA) is a promising way to solve combinatorial optimization problems [1]. In the optimization problems, we need to minimize a cost function, and some of the optimization problems can be mapped into a task to find a ground state of the Ising Hamiltonian [2]. QA is designed to find such a ground state of the Ising Hamiltonian by using adiabatic dynamics [3].

Most of the previous demonstrations of the QA has been done with the superconducting flux qubits (FQ) [4]. Since the FQ is an artificial atom, there are many degrees of design freedom. We can control the properties of the qubit by changing the circuit design before the fabrication. Also, even after the fabrication, control line coupled with each qubit can change the qubit frequency and qubit-qubit coupling strength. These are prerequisite for the QA. On the other hand, the FQ used for the QA has a short coherence time such as nano seconds [5]. To exploit the quantum advantages, it should be better to use the qubits with better coherence time.

Recently, a capacitively-shunted flux qubit (CSFQ) was demonstrated. The CSFQ has a coherence time of an order of tens of micro seconds at near the optimal point [6], and this is a few orders of magnitude larger than that of the FQs used in the previous QA. So the CSFQ is considered as a promising candidate to realize quantum information processing.

However, in order to use the CSFQ for the QA, there are two main problems to overcome. Firstly, the CSFQ has a smaller persistent current such as tens of nA [6] while the standard FQ has a persistent current of a few μA [4]. This results in a few order of magnitude smaller coupling strength between qubits. To solve practically useful problems with the QA, the coupling strength of the qubits should be comparable with the qubit frequency, but it would be difficult to achieve such a strong coupling in the CSFQ. Secondly, near the optimal point where the coherence time is maximized, the CSFQ has not only Ising interaction but also flip-flop interaction with another CSQS [6]. The QA exploits the Ising interaction to solve the problems, and the residual flip flop type interaction could induce a fatal error to find the solutions.

In this paper, we propose to implement the spinlock-based QA with the CSFQs. The spin lock technique is designed to keep a state of $|+\rangle$ (an eigenstate of σ_x) in a rotating frame. More specifically, after we prepare a state of $|+\rangle$ by performing a $\pi/2$ pulse, we continuously drive the qubit along x direction, which keeps the state in $|+\rangle$ that is an eigenstate of the Hamiltonian in the rotating frame. This technique has been widely used in the fields of magnetic resonance [7]. Importantly, when the qubit is driven by the AC fields, the effective qubit frequency becomes the detuning between the qubit frequency and driving field frequency. Moreover, a large detuning between the bare frequencies of the qubits can be set during the spin lock on every qubit, and this detuning effectively suppresses the flip-flop interaction between the qubits. Although such a spin-lock based QA has been implemented with NMR [8,9], a practical benefit is unclear because it is difficult to increase the number of the qubits in NMR. On the other hand, we propose to implement the QA with CSFQs that are expected to have a scalability.

We theoretically investigate the performance of the spin-lock based QA with the CSFQs. Especially, the spin-lock based QA becomes equivalent to the conventional QA only if the rotating wave approximation (RWA) is valid. When we drive the qubits with strong driving fields, the RWA can be violated when the qubit bare frequency becomes close to the Rabi frequency. In NMR, the frequency of the qubit is 6 orders of magnitude larger than the other typical frequencies [8], the RWA is quite accurate. However, the frequency of the CSFQ is just a few orders of magnitude larger than the other frequencies. So careful assessment of the error accumulation is required to investigate the practicality of the spin-lock based QA with CSFQs.

2. The standard QA with DC transverse magnetic fields

Before we explain our QA by using the system with the Heisenberg interaction, let us quickly review the standard QA with applying DC transverse magnetic fields [1]. The Hamiltonian in the QA is described as follows.

$$H_{\mathrm{QA}} = e^{-\gamma^2 t^2} H_{TR} + (1 - e^{-\gamma^2 t^2}) H_{\mathrm{Ising}} \qquad (1)$$

$$H_{\text{Ising}} = -\sum_{i=1}^{L} \frac{h_i}{2} \sigma_z^{(i)} - \sum_{i,i'=1}^{L} \frac{J_{i,i'}}{2} \sigma_z^{(i)} \sigma_z^{(i')} \quad (2)$$

$$H_{\text{TR}} = \sum_{i=1}^{L} \frac{\Lambda}{2} \sigma_x^{(i)} \quad (3)$$

where γ denotes the typical time scale to reduce the transverse fields, h_i denotes the resonant frequency of the i-th qubit, $J_{ii'}$ denotes the coupling strength between the qubits, and Λ denotes the amplitude of the transverse magnetic fields. For the QA, we prepare a state of $|+\rangle$ for all qubits, and we reduce the amplitude of the transverse magnetic fields with a time scale of γ while we adiabatically increase the amplitude of H_{Ising} with the same time scale. If γ is much smaller than the energy gap between the ground state and first excited state of H_{QA} for all t, we can obtain a ground state of the Ising Hamiltonian.

3. QA with spin lock technique

We explain the details of the QA with the spin lock technique [12,13]. Firstly, we prepare a spin down state (an eigenstate of σ_z) for every qubit. Secondly, we apply a global $\pi/2$ pulse to prepare a state of $|+\rangle$. Thirdly, we continuously drive the qubit along x direction, and gradually reduces the amplitude of the transverse driving field while we gradually turn on the Ising Hamiltonian. Finally, we readout the qubits. The third step in the scheme is governed by the unitary evolution based on the following Hamiltonian

$$H = H_0 + e^{-\gamma^2 t^2} H_D + (1 - e^{-\gamma^2 t^2})(H_{\text{Ising}} + H_{\text{xy}}) \quad (4)$$

$$H_0 = \sum_{i=1}^{L} \frac{\omega + \delta\omega_i}{2} \sigma_z^{(i)} \quad (5)$$

$$H_D = \sum_{i=1}^{L} \lambda \cos[(\omega + \delta\omega_i)t] \sigma_x^{(i)} \quad (6)$$

$$H_{\text{xy}} = -\sum_{i,i'=1}^{L} \frac{J_{i,i'}}{2} \sigma_x^{(i)} \sigma_x^{(i')} - \sum_{i,i'=1}^{L} \frac{J_{i,i'}}{2} \sigma_y^{(i)} \sigma_y^{(i')} \quad (2)$$

where $(\omega + \delta\omega_i)$ denotes a bare frequency of the i-th qubit and λ denotes the Rabi frequency of the driving fields. Importantly, by taking a rotating wave approximation (RWA) that becomes valid in the limit of a large qubit frequency, the Hamiltonian in the Eq (4) becomes equivalent to that in the Eq. (1) by setting $\lambda = \Lambda$.

5. Possible implementation of the practical QA with the spin-lock technique

We evaluate the potential errors during the spin-lock based QA due to the violation of the RWA by using numerical simulations. We solve a time-dependent Schrödinger equation with Hamiltonian (4) from $t = 0$ to $t = T$ where the initial state of every qubit is $|+\rangle$. We consider a ferromagnetic one-dimensional Ising model with free boundary condition. More specifically, we consider $h_j = h > 0$ and $J_{i,i+1} = J > 0$ (i=1,2, $\cdots$, L-1) with the nearest neighbor interaction. In this case, the ground state of the Ising Hamiltonian is an all up state of $|11\cdots1\rangle$. To quantify the accuracy to find the ground state of the Ising Hamiltonian, we define a fidelity as $F(t) = |\langle 11\cdots1|\phi(t)\rangle|^2$ where $|\phi(t)\rangle$ denotes a solution of the Schrödinger equation at a time t. We plot an infidelity (1-F) against time with feasible parameters of the CSFQs in the Fig. 1. The fidelity increases as the qubit frequency increases. This shows that the larger frequency of the qubit makes the RWA more valid, which reduces the error in the QA.

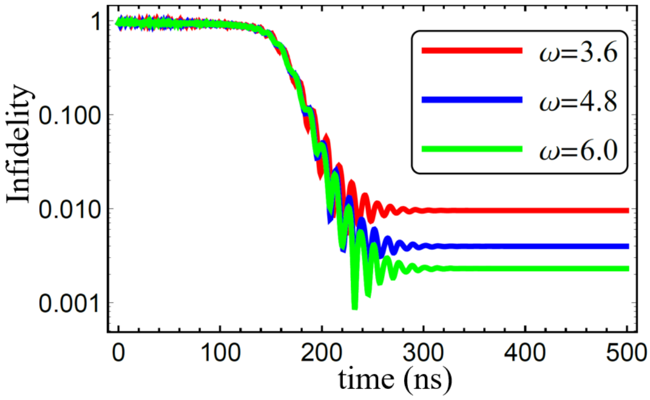

Fig. 1 Infidelity plotted against time. Here, we set parameters as $\lambda/2\pi$=1 GHz, $h/2\pi$=0.03 GHz, γ = 0.01 GHz, $J/2\pi$=0.05, T=500 (ns), $\omega/2\pi$=3.6, 4.8, 6.0 GHz, $\delta\omega_1/2\pi$=0, $\delta\omega_2/2\pi$=3.7, $\delta\omega_3/2\pi$=-0.5, L=4, and $\delta\omega_4/2\pi$=3, which are typical in the SCFQ [10].

5. Conclusions

In conclusion, we propose to implement the QA with CSFQs. Although it is difficult to perform the conventional QA by using the CSFQs because of the weak coupling strength and residual flip-flop interactions, we show that a use of the spin-lock based QA can overcome these problems. Our numerical simulations show that the spin-lock based QA can be implemented even with the current technology.

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