PS-9-13 (Late News) Extended Abstracts of the 2019 International Conference on Solid State Devices and Materials, Nagoya, 2019, pp953-954

Single Spin Detection With Entangled States

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Abstract

Single spin detection is one of the important tasks in quantum metrology. Many experiments about the single spin detection has been performed. However, due to the weak magnetic fields from the single spin, a long measurement time is required to achieve a reasonably high signal-to noise-ratio, and so a practical single spin detection is still challenging in the current technology. Here, we propose an alternative way to realize rapid and accurate single spin detection with entangled states. Although entanglement is known to be resource to improve the sensitivity for measuring global magnetic fields, it is not clear how entanglement can be used to detect spatially inhomogeneous magnetic fields, which is the case of the single spin detection. We theoretically show that the entanglement significantly increases the signal to noise ratio for the single spin detection even under the effect of realistic noise. Our results pave the way for practical spin detection.

1. Introduction

One of the ultimate goals in quantum metrology [1] is the detection of weak magnetic fields from the single (electron or nuclear) spin [2-12]. The single spin detection has been considered as an attractive technique due to the potential applications in various areas such as material science and biology. Extensive research has been conducted, including both theoretical and experimental approaches. Single spin detection requires a magnetic field sensor that has not only the high sensitivity but also an excellent spatial resolution. However, the conventional sensors typically suffer from a trade-off relation between the sensitivity and spatial resolution. The conventional magnetic field sensor with a spatial resolution to detect a single spin has a worse sensitivity than the other larger sensors. This results in a long measurement time to compensate the low signal to noise ratio. So, the improvement of the sensitivity of the sensor without sacrificing the spatial resolution is essential for more efficient single spin detection.

In this work, we propose the single-spin sensor using a specific type of entanglement, called GHZ states. The GHZ states are famous not only as “Schrödinger cat states”, introduced in the context of a fundamental issue on quantum physics, but also as resource to improve the sensitivity to measure global magnetic fields [13-15]. However, single spin detection with the GHZ states is not a trivial extension of the case of global magnetic field sensors. Because of the dipole-dipole interaction between the target (single) spin and the probe spins (GHZ states), this entangled sensor needs to detect the spatially inhomogeneous magnetic field. Especially, magnetic fields on the probe spins far from the target spin are much smaller than that of probe spins close to the target spin. This means that the probe spins far from the target spin just induce noise without contributing the increase of the signal. By considering these conditions, we investigate the optimal number of the probe spins, and we find that the magnetic sensor with the GHZ states achieves a few orders of magnitude better sensitivity than that with the single spin or the separable spin ensemble even under the effect of realistic noise.

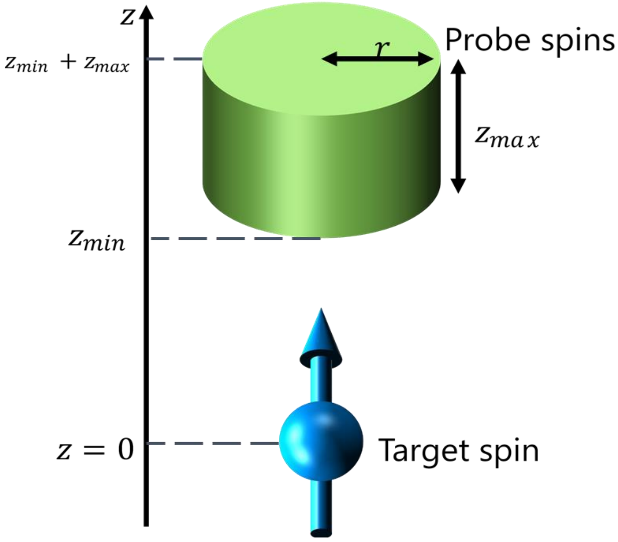

Fig. 1: This illustrates the positional relation between the target spin and the probe spins. The probe spins are homogeneously distributed inside a columnar substrate.

2. Setup and Models

We explain the setup of our calculation. The configuration of the target spin and ensemble of probe spins is shown in the Fig. 1. There are L probe spins homogeneously distributed with the density of ρ inside a columnar substrate. The effective Hamiltonian due to dipole-dipole interaction between the target spin and the probe spins in the rotating frame with the rotating approximation is given by

$$\hat{H}^{(\mathrm{eff})} = \sum_{j=1}^{L} \frac{G}{\left(r_j^2 + z_j^2\right)^{3/2}} \left(\frac{3z_j^2}{r_j^2 + z_j^2} - 1\right) \sigma_z^{(T)} \sigma_{z,j}^{(P)}, \quad (1)$$

where G is the product of the magnetic moments of the target spin and the probe spins, and the coordinate (x, y, z) is defined as Fig.1, and we set the cylindrical coordinate $r = \sqrt{x^2 + y^2}$. $\sigma_{z,j}^{(P)}$ is the Pauli Z matrix at (x_j, y_j, z_j). Since we consider a case either the target spin is up or down, we replace $\sigma_z^{(T)}$ with a classical value s=1 or -1. The variations of the distance between the target spin and each probe spin causes the inhomogeneous magnetic fields acting on the probe spins. We will consider either a single spin, an ensemble of separable spins [16], or entangled spins, as the probe states. When we consider the single probe spin, the probe spin is attached at the center bottom of the columnar substrate. As an entanglement, we choose the GHZ state:

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\cdots\uparrow\rangle + |\downarrow\downarrow\cdots\downarrow\rangle)\,. \quad (2)$$

The time evolution of the quantum states under the effect of non-Markovian dephasing is described by the master equation [13]

$$\frac{\partial\hat{\rho}(t)}{\partial t} = -i\left[\hat{H}^{(\mathrm{eff})},\hat{\rho}(t)\right] - \frac{t}{4T_2^2}\sum_{j=1}^{L}\left[\hat{\sigma}_{z,j},\left[\hat{\sigma}_{z,j},\hat{\rho}(t)\right]\right], \quad (3)$$

where $\hat{\rho}(t)$ is the density operator at time t and T_2 denotes the coherence time of the qubit.

We describe our measurement sequence. Firstly, prepare an initial state (shown in Table I). Secondly, let the quantum state evolve by Eq. (3) for a time *t*. Thirdly, measure the state by a specific readout basis (shown in Table I). Finally, we repeat 1-3 steps N times. We assume that the state preparation time and readout time is negligibly small, and we can approximate $N \approx T/t$ where T is a given total measurement time.

Table I Preparation of initial states and readout basis

	Initial states	Readout basis
Single spin	$\|+\rangle$	$(\|\uparrow\rangle \pm i\|\downarrow\rangle)/\sqrt{2}$
Ensemble spins (separable)	$\|++\cdots+\rangle$	$\otimes_j\,[(\|\uparrow\rangle_j \pm i\|\downarrow\rangle_j)/\sqrt{2}]$
Entangled spins	$\|\mathrm{GHZ}\rangle$	$(\|\uparrow\cdots\uparrow\rangle \pm i\|\downarrow\cdots\downarrow\rangle)/\sqrt{2}$

3. Results

We calculate the uncertainty of the estimation by optimizing r and $z_{\max}$. Here, the inverse of the uncertainty corresponds to the sensitivity. The results are summarized as the below Table II. It is worth mentioning that, while the global field B is estimated in the previous studies, we estimate the value of s of the target spin.

Table II Summary of results

	Global B	Single spin detection
Single spin	$\delta B = O(L^0)$	$\delta s^{(\mathrm{s})} = \frac{\sqrt{2}e^{1/4}}{4G\sqrt{TT_2}} z_{min}^3$ [16]
Ensemble spins (separable)	$\delta B = O(L^{-1/2})$ (standard quantum limit)	$\delta s^{(\mathrm{sep})} = \frac{5.32\sqrt{2}e^{1/4}}{4G\sqrt{\pi}\sqrt{TT_2}}\frac{z_{min}^{3/2}}{\sqrt{\rho}}$ [16]
Entangled spins	$\delta B = O(L^{-3/4})$ [13-15]	$\delta s^{(\mathrm{en})} = \frac{4.14\sqrt{2}e^{1/4}}{4G\pi^{3/4}\sqrt{TT_2}}\frac{z_{min}^{3/4}}{\rho^{3/4}}$ (our results)

Numerical simulations

We use realistic parameters for the nitrogen vacancy (NV) centers in diamond as probe spins, which is considered as a promising device on quantum information processing, and numerically investigate the uncertainty of the estimation. Fig. 2 shows the ratio of the uncertainty δs against $z_{\min}$. Here, we choose $\rho = 6.7\times10^{16}/\mathrm{cm}^3$ for an ensemble of the NV centers, $T_2 = 2\mathrm{ms}$ (single spin), and $T_2 = 84\mu\mathrm{s}$ (ensemble spins with separable or entangled states). From this graph, we find that the ratio of the uncertainty is $\delta s^{(\mathrm{s})}/\delta s^{(\mathrm{en})} \simeq 500$ at $z_{\min} = 1\ \mu\mathrm{m}$, which means that the probe state with the entangled spins achieve 500 times higher sensitivity than that of the single spin. On the other hand, for the probe state with the separable spins, the ratio is $\delta s^{(\mathrm{s})}/\delta s^{(\mathrm{sep})} \simeq 17$ at $z_{\min} = 1$, and this also achieves higher sensitivity but is not so sensitive as that of the entangled spins. Moreover, with the entangled spins, the ratio is over 1 in the larger range. From the Fig.2 (inset), we can see that $\delta s^{(\mathrm{s})}/\delta s^{(\mathrm{sep})} > 1$ at $z_{\min} > 0.15\ \mu\mathrm{m}$, while $\delta s^{(\mathrm{s})}/\delta s^{(\mathrm{en})} > 1$ at $z_{\min} > 0.065\ \mu\mathrm{m}$.

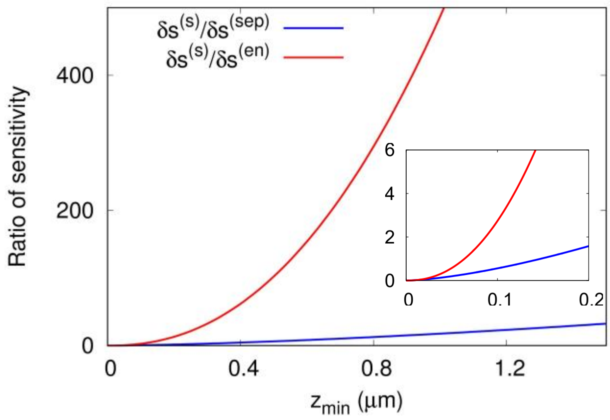

Fig. 2: This graph depicts the ratio of the uncertainty against $z_{\min}$. (Inset) the enlargement of the graph.

4. Conclusion

We have shown that entangled spins are remarkably useful resources to detect the target single spin. We expect that by using entangled states, more efficient detection of single spin will be achieved experimentally near the future.

Acknowledgements

We are grateful to Syuhei Uesugi, S. Endo, and Jumko Hayase for their assistant in this study. This work was supported by Leading Initiative for Excellent Young Researchers MEXT Japan, and is partially supported by MEXT KAKENHI (Grant No. 15H05870).

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