# A Study of Dynamics of Skyrmioniums by Spin Transfer Torque

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### **Abstract**

We investigate the dynamics of skyrmioniums by spin transfer torque using micromagnetic simulation. Generally, as for skyrmions, the skyrmion Hall effect occurs depending on the sign of the topological charge when the Gilbert damping constant is not equal to the non-adiabatic constant. However, it is found that the skyrmionium Hall effect does not occur since its topological charge is zero. This result suggests that skyrmioniums are more suitable for memory devices or logic ones than skyrmions.

### 1. Introduction

Recently, magnetic skyrmions have been paid much attention due to potential ability for high-density memories and logic devices in the field of spintronics [1]. This is because skyrmions are relatively stable against perturbations and can be driven by a very low current density in comparison with a critical current density for domain wall motion. The stability of skyrmions originates from the fact that they are topological solitons. However, there exists a potential disadvantage for us to utilize skyrmions for spintronics devices. The disadvantage is that it is difficult to manipulate skyrmions using an electrical current since skyrmions are deflected against the direction of the electrical current flow, which is called the skyrmion Hall effect. This is attributed to the fact that skyrmions have topological invariants which are called topological charges Q and that the topological charges of skyrmions have the values of either Q = +1 or Q = -1. Therefore, it is expected that we can overcome the above disadvantage using magnetic structures whose topological charges are Q = 0.

Magnetic skyrmioniums have vortex-like structures which are equivalently composed of  $Q=\pm 1$  skyrmions and the topological charges of skyrmioniums are Q=0. Therefore, it is speculated that skyrmioniums move along the electrical current flow. In our study, we investigate the dynamics of skyrmioniums caused by the spin transfer torque (STT) using the micromagnetic simulation.

## 2. Theory

Magnetization of an isolated skyrmionium can be described by the spherical coordinates as follows:

$$m = (\cos \Phi(\phi) \sin \theta(r), \sin \Phi(\phi) \sin \theta(r), \cos \theta(r)),$$
 (1)

where m is the magnetization unit vector and  $\Phi(\phi) = n\phi + \gamma$ . Here,  $\phi$  is the azimuth angle, n is the vorticity, and  $\gamma$  is the helicity. We consider the case of n=1 and  $\gamma=0$  in this study. In order to obtain the skyrmionium structure, we utilize the variational principle and find the lowest energy state of the magnetization given by eq. (1) [2, 3]. As a result, we derive the following Euler-Lagrange equation of the angle of  $\theta$ :

$$\frac{\mathrm{d}^2 \theta}{\mathrm{d}\tilde{r}^2} + \frac{1}{\tilde{r}} \frac{\mathrm{d}\theta}{\mathrm{d}\tilde{r}} - \frac{\sin\theta\cos\theta}{\tilde{r}} + \frac{D}{\sqrt{AK}} \frac{\sin^2\theta}{\tilde{r}} - \sin\theta\cos\theta = 0, (2)$$

where A is the exchange stiffness, K is the perpendicular anisotropy constant, D is the Dzyaloshinskii-Moriya interaction (DMI) constant, and  $\tilde{r} = r\sqrt{K/A}$ . In order to solve eq. (2) for obtaining the magnetization of the skyrmionium, the conditions of  $\theta(0) = 2\pi$  and  $\theta(\infty) = 0$  are imposed for the boundary condition.

The skyrmionium structure derived from eq. (2) has the topological charge which is topological invariant which is defined by

$$Q = \frac{1}{4\pi} \int \boldsymbol{m} \cdot \left( \frac{\partial \boldsymbol{m}}{\partial x} \times \frac{\partial \boldsymbol{m}}{\partial y} \right) dx dy.$$
 (3)

This corresponds to how many times the magnetization wraps the 2-sphere ( $S^2$ ). Inserting eq. (1) to the above equation, the topological charge of the skyrmionium becomes Q = 0.

When the STT is applied to the skyrmionium, the magnetization dynamics is governed by the modified Landau-Lifshitz-Gilbert (LLG) equation which is written by

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mu_0 \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} + \frac{\tau}{M_c}, \quad (4)$$

where  $\gamma$  is the gyromagnetic ratio,  $\mu_0$  is the vacuum permeability,  $\alpha$  is the Gilbert damping constant,  $M_s$  is the saturation magnetization,  $\boldsymbol{H}_{\text{eff}}$  is the effective magnetic field,  $\boldsymbol{\tau} = \frac{\gamma\hbar P}{2e(1+\beta^2)} [(\boldsymbol{j}\cdot\nabla)\boldsymbol{m} - \beta\boldsymbol{m}\times(\boldsymbol{j}\cdot\nabla)\boldsymbol{m}]$  is the STT [4]. Here,  $\hbar$  is the Dirac constant, P is the spin polarization, e is the electron charge,  $\boldsymbol{j}$  is the current density, and  $\beta$  is the non-adiabatic parameter.

Steady state motion of magnetic structures is represented by the Thiele equation [5]. The Thiele equation is derived from the LLG equation (4) and is written as follows:

$$G \times \left( v - \frac{\gamma \hbar P}{2eM_s(1+\beta^2)} j \right) - 4\pi \alpha \mathcal{D} \cdot \left( v - \frac{\gamma \hbar P}{2eM_s(1+\beta^2)} \frac{\beta}{\alpha} j \right) = 0, \quad (5)$$

where  $G=(0,0,-4\pi Q)$  is the gyromagnetic coupling vector, v is the velocity of isolated magnetic structures like magnetic vortices, skyrmions, skyrmioniums, etc., and  $\mathcal{D}=\begin{pmatrix} \mathcal{D} & 0 \\ 0 & \mathcal{D} \end{pmatrix}$  is the dissipative force tensor. Here, the components of this tensor  $\mathcal{D}$  are determined by isolated magnetic structures. When the electric current density has only x-component like  $j=(j_x,0)$ , the Thiele equation (5) provides the velocity components of isolated magnetic structures as follows:

$$v_{x} = \frac{-Q^{2} - \alpha\beta\mathcal{D}^{2}}{Q^{2} + \alpha^{2}\mathcal{D}^{2}} \frac{\gamma\hbar P}{2eM_{s}(1+\beta^{2})} j_{x},$$

$$v_{y} = \frac{Q\mathcal{D}(\beta - \alpha)}{Q^{2} + \alpha^{2}\mathcal{D}^{2}} \frac{\gamma\hbar P}{2eM_{s}(1+\beta^{2})} j_{x}.$$
(6)

Equations (6) lead to the result that the motion of any isolated magnetic structures is parallel with the electric current whenever the condition of  $\alpha=\beta$  holds. On the other hand, the motion of skyrmioniums is always parallel with the electric current regardless of the values of  $\alpha$  and  $\beta$  since the topological charge of the skyrmionium is Q=0. Although we can obtain the above significant result from the Thiele equation, it is mandatory to solve the LLG equation numerically. This is because the result obtained from the Thiele equation is not always identical to the one obtained from the LLG equation [6].

### 3. Results and discussion

In micromagnetic simulation, we utilize the following material parameters:  $M_s = 600 \text{ kA/m}$ , A = 30 pJ/m,  $D = 4 \text{ mJ/m}^2$ ,  $K = 0.6 \text{ MJ/m}^3$ ,  $\alpha = 0.2$ ,  $\beta = 0.4$ , and P = 0.4. Then, the skyrmionium structure can be obtained using variational method as shown in Fig. 1. This figure shows the skyrmionium is equivalently composed of  $Q = \pm 1$  skyrmions. In order to obtain a size of the skyrmionium, we define a skyrmionium radius as a point where the angle of  $\theta(r)$  becomes  $2\pi/e$ . Then, the radius of the skyrmionium is 60.2 mm

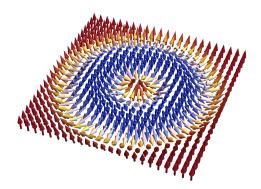


Fig. 1 A structure of skyrmionium obtained by the variational principle. The direction and the color of arrows represent magnetization-direction and the magnitude of *z*-component of magnetization at each point, respectively.

Next, we utilize the above skyrmionium for the initial state of the micromagnetic simulation. In the simulation, we apply the current density  $j_x = -1 \text{ TA/m}^2$  in the *x*-direction. The snap shot of time evolution of the calculated skyrmionium motion

is shown in Fig. 2. The result of the simulation shows that the skyrmionium certainly moves in parallel with the electric current. On the other hand, the result also shows that the motion is fluctuating slightly around the straight line. This result is almost consistent with that obtained from the Thiele equation. However, we have found that the motion of the skyrmionium is not exactly straight and that the motion is like zigzag line from the detailed viewpoint. This result will be discussed in detail in the conference. Using the result of the simulation, we obtain the time averaged skyrmionium velocity components of  $\langle v_x \rangle = 72.3$  m/s and  $\langle v_y \rangle = 0.5$  m/s during the time period of 5 ns.

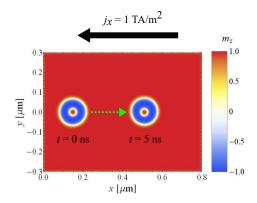


Fig. 2 The snap shot of time evolution of the calculated skyrmionium motion. The green dotted line is a guide for eyes.

### 4. Conclusion

It is found that skyrmioniums can also be manipulated by the STT like skyrmions. Moreover, the motion of skyrmioniums is always parallel with the electric current unlike skyrmions even though the Gilbert damping constant  $\alpha$  is not equal to the non-adiabatic parameter  $\beta$  of the STT since the topological charge of skyrmioniums is equal to zero. This fact suggests that skyrmioniums can be more easily manipulated by the STT than skyrmions. Therefore, it is considered that skyrmioniums are more suitable for memory devices or logic ones than skyrmions.

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