Estimation of Phase Velocity using an Array with Arbitrary Shape

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1. Introduction
Microtremor observation has been widely applied for estimating ground structures. Two methods have been generally recognized to calculate phase velocities, which are the frequency-wave number (F-K) spectral method (Capon, 1969) and the spatial auto-correlation (SPAC) method (Aki, 1957). Although F-K method has no constraints on the shape of the array, the accuracy of results depends on the array shape. In addition, the SPAC method requires particular arrangement of the sensors, which can be difficult to realize in field observation. Attempts, such as Centerless Circular Array (CCA) method (Cho et al., 2006) has been made to eliminate constraints on the shape of array. It can be applied to arbitrary array shape considering sensors spacing around a specific circle, but the determining process for which can be complex. Therefore, a method of estimating phase velocity of Rayleigh wave using an arbitrary shape array is proposed.

2. Method
On the basis of the analytical solution of Lamb's problem for vertical components of Rayleigh wave, the relationship between two observation points, p and q for example, is given in a discrete representation with the Bessel function of the first kind and higher-order Bessel functions, which is called Complex Coherence Function (CCF; 白石・松岡, 2005). Zhang and Morikawa (2015) extended the CCF to apply it to linear array situation. By adding an observation point s, which is located anticlockwise alpha degree with respect to line p-q, the relationship between p and s can be expressed as equation (1), in which the left side denotes the real part of the normalized cross spectra of vertical records between point p and s, and character Jn denotes the n-th order Bessel function of the first kind. The r_ps is the distance between point p and s. k is the wave number, which can be expressed as $2\pi f/c$. Here, c is unknown phase velocity. And the Xn and Yn are defined as sum of the m-th source's power ratio multiplied by cosine and sine of $2n\theta$. θ denotes the source direction. (盛川・飯山, 2015).
For kr of range $[0, \pi]$, compared with lower-order Bessel functions' value, Bessel functions of order greater than 6 can be ignored. Hence, the CCF equation have only 5 unknowns left, which are c, X1, Y1, X2, and Y2. In order to find the unknown parameters, Artificial Bee Colony (ABC) algorithm (Karaboga and Basturk, 2007) is applied.

3. Problem Setting and Results
Four shapes of array are considered as shown in Figure 1(a). Numerical simulation was conducted by propagating randomly generated wave sources and thus obtain the cross spectra between different sites, where the power spectral density function is defined as equation (2), in which A is the amplitude and D is the denominator. For every source, the amplitude, denominator and source direction will be randomly chosen from set {1,2,4,8}, set {2,4,5,8,10} and set $\{\pi/18, \pi/9, \pi/6, \ldots, 35/18\pi, 2\pi\}$. The results with array size L=100m and 8 random sources are shown in Figure 1(b). It shows that the estimation results of sources with random directions and power conditions match the theoretical value for frequency range from 0.25 Hz to 1.5 Hz for case 1 and 2. However, the accurate range narrows for case 3 but the result is still acceptable. For case 4, a linear array, the estimation is relatively unsatisfactory. In conclusion, except

for the linear array situation, it is available to estimate the phase velocity of Rayleigh wave using arbitrary array shape.

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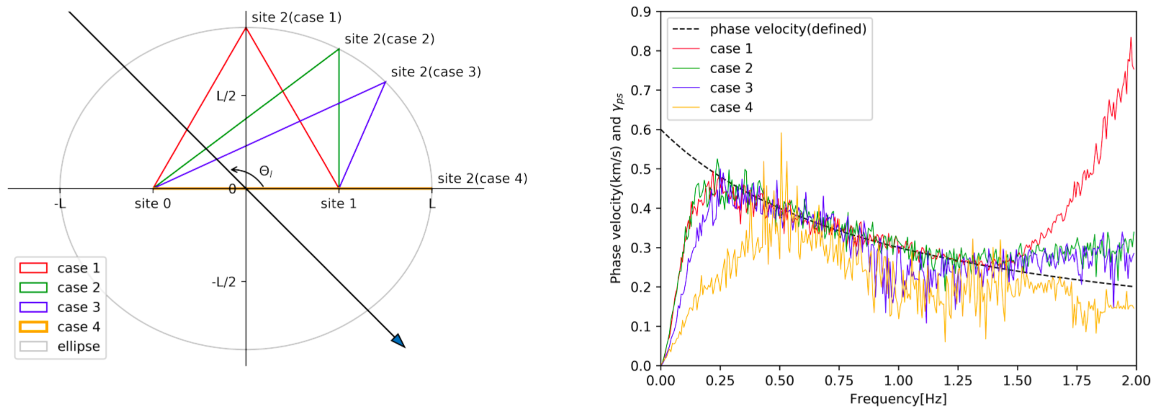
$$\Re[\gamma_{ps}] = J_0(kr_{ps}) + 2\sum_{n=1}^{\infty}\{(-1)^n J_{2n}(kr_{ps})(X_n cos2n\alpha + Y_n sin2n\alpha)\}, \qquad (1)$$

$$PSD = \begin{cases} A \cdot |sin\frac{2\pi f}{D}|, & 0 \le f \le 10Hz \\ 0, & 10Hz < f \le 50Hz, \end{cases} \qquad (2)$$



(a) Array setting

(b) phase velocity of ABC

Figure 1: Array setting and phase velocity result with ABC algorithm