Hologram Calculation of Light-in-flight Recording by Holography based on Numerical Simulation Model with FDTD Method

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ABSTRACT

We propose a numerical simulation model to calculate a hologram of light-in-flight recording by holography. The proposed model is based on not only ray tracing but also finite-difference time-domain method. We succeeded in numerically reconstructing light pulse propagation with total reflection from the hologram calculated by the proposed model.

1 INTRODUCTION

In cutting-edge ultrafast photonics, techniques which enables us to realize observation and visualization of light pulse propagation are useful for elucidation of dynamics and mechanism of nonlinear and ultrafast phenomena. Some techniques have been proposed and demonstrated: femtosecond time-resolved optical polarigraphy [1], photon scanning tunneling microscopy [2], femto-photography [3], time-correlated single-photon counting [4], and light-in-flight recording by holography (LIF holography) [5]. We focus on LIF holography because it has several attractive features as follows. LIF holography enables us to record and reconstruct light pulse propagation as a moving pictures which are spatially and temporally continuous. In addition, moving pictures can be obtained with single light pulse with single-shot exposure in principle. Furthermore, LIF holography can record and reconstruct not only two-dimensional but also three-dimensional moving pictures [6]. In LIF holography, numerical simulation model is important for analyzing and verifying experimental results. However, conventional simulation models were based on only ray tracing [7]. In other words, the difference of optical-path length between object wave and reference wave was considered in the model. Then, it is difficult for the conventional models to simulate light propagation of complex situations such as refraction, diffraction, total reflection, polarization changes, and so on [8]. As a result, the conventional simulation models have the major problem that analyzable situations are strictly limited. In this work, therefore, we aim to overcome the limitation by applying finite-difference time-domain (FDTD) method [9] to the numerical simulation of light-in-flight recording by holography. We propose a novel simulation model for LIF holography, and calculate a hologram of light-in-flight recording by holography.

2 Principle

2.1 Light-in-flight recording by holography

Fig. 1(a) shows the schematic diagram of top view of an optical setup in recording of LIF holography. A light pulse emitted from an ultrashort pulsed laser is divided into two pulses by a beam splitter. These pulses are collimated by beam expanders. One pulse called an illumination pulse is incident on the diffuser plate, and generates a scattered pulse. A light pulse scatted by the diffuser plate is called an object pulse. Here, the object pulse is generated from horizontally different position at different time because the illumination pulse is introduced into the diffuser plate at an oblique angle. On

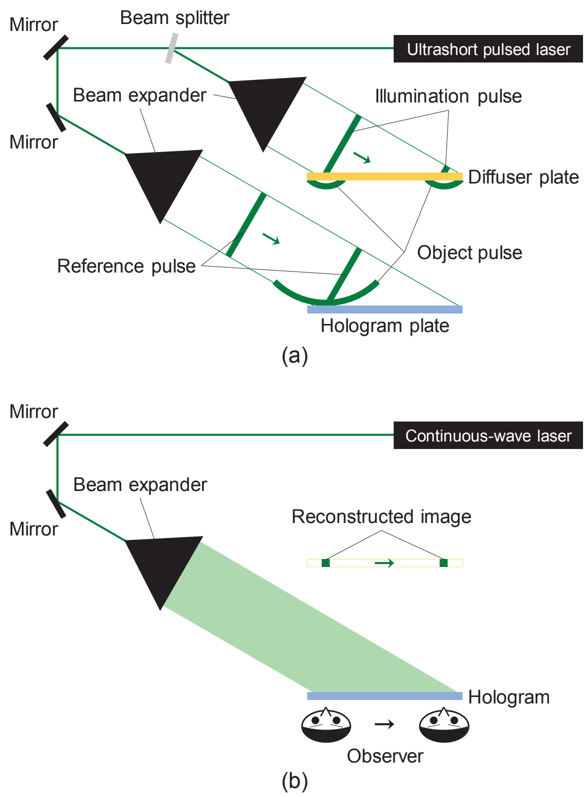


Fig. 1 Schematic diagram of top view of optical setup in LIF holography. (a) Recording. (b) Reconstruction.

the other hand, a pulse called a reference pulse is incident on the hologram plate at an oblique angle. Then, because the temporal coherence of the ultrashort light pulse is low, interference fringes are formed by the object and reference pulses only when they arrive at the hologram plate at the same time. Moreover, the reference pulse arrives at horizontally different position on the hologram plate at different time. Therefore, temporal information of the object pulse is recorded on the hologram plate as spatial information in horizontal direction in LIF holography.

Fig. 1(b) shows the schematic diagram of top view of an optical setup in reconstruction of LIF holography. We illuminate the recorded hologram by continuous-wave light at the same angle as the reference pulse. Then, holographic images are reconstructed from the hologram. Here, to obtain spatially and temporally continuous moving pictures of light pulse propagation from the hologram, we need to move the gazing point on the hologram horizontally. This is because different temporal information of the object pulse is recorded at different position of the hologram along horizontal direction.

2.2 Computer-generated hologram

A computer-generated hologram (CGH) is obtained by numerical simulation on a computer by calculating light propagation and interference. Although many algorithms have been proposed for CGH calculation, we adopted the point-cloud method [10] to calculate a kinoform which is a phase-modulation-type CGH in this work.

2.3 Finite-difference time-domain method

FDTD method is one of the algorithms for electromagnetic field analysis. It solves Maxwell's equations using differential equations in time domain. In FDTD method, time-dependent Maxwell's equations are discretized by using central-difference approximations. In FDTD method, the electric-field components are positioned in the middle of the edge of the grid, whereas the magnetic-field components are positioned in the center of the faces of the grid. The detail of the discretized differential equations in FDTD method are explained in Ref. [9].

3 Numerical simulation model of LIF holography

3.1 Conventional model without FDTD method

Fig. 2 shows the schematic diagram of the conventional simulation model of LIF holography [7]. For simplicity, we assume the same optical setup as Fig. 1(a). A hologram is set on xy plane. The diffuser plate is parallel to the hologram plane and is positioned at $z = d$. We define parameters required for the simulation model as shown in Table 1. We assume that the object pulse generated at $\mathrm{D_O}(0, 0, d)$ and the reference pulse simultaneously arrive at $\mathrm{O}(0, 0, 0)$ at $t = 0$. We also assume that point-light sources are distributed on the diffuser plate. Then, we investigate whether object pulse from each point-light source can be recorded at $\mathrm{H_{mn}}(x_m, y_m, 0)$ or not. First, we consider $T_{\mathrm{D_O \to O}}$ representing the time it takes the object pulse emitted from $\mathrm{D_O}$ to propagate to O. Because the distance between $\mathrm{D_O}$ and O is d, $T_{\mathrm{D_O \to O}}$ is given by

$$T_{\mathrm{D_O \to O}} = d/c\,, \tag{5}$$

where c indicates the velocity of light. Then, by defining $t_{\mathrm{D_O}}$ as the time when the illumination pulse reaches $\mathrm{D_O}$, we obtain the following equation:

$$t_{\mathrm{D_O}} = -T_{\mathrm{D_O \to O}}\,. \tag{6}$$

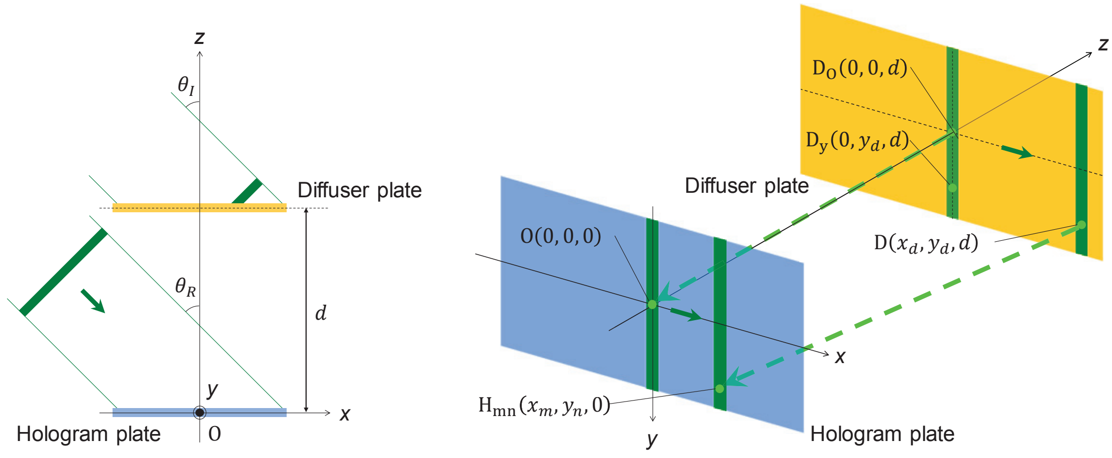
Secondary, we consider $T_{\mathrm{D_y \to D}}$ representing the time it takes the illumination pulse propagates from $\mathrm{D_y}(0, y_d, d)$ to $\mathrm{D}(x_d, y_d, d)$ on the diffuser plate. $T_{\mathrm{D_y \to D}}$ is given by

$$T_{\mathrm{D_y \to D}} = x_d \sin\theta_I / c\,. \tag{7}$$

Because the illumination pulse reaches $\mathrm{D_y}$ at $t_{\mathrm{D_O}}$, we obtain the following equation by defining t_D as the time when the illumination pulse reaches D:

$$t_{\mathrm{D}} = t_{\mathrm{D_O}} + T_{\mathrm{D_y \to D}}\,. \tag{8}$$

Thirdly, we consider $T_{\mathrm{D \to H}}$ representing the time it takes



(a) (b)

Fig. 2 Schematic diagram of conventional simulation model in LIF holography. (a) Top view. (b) Bird's eye view.

the object pulse emitted from D propagates to $\mathrm{H_{mn}}$. $T_{\mathrm{D\to H}}$ is given by

$$T_{\mathrm{D\to H_{mn}}} = \overline{\mathrm{DH_{mn}}}/c = \sqrt{(x_m - x_d)^2 + (y_n - y_d)^2 + d^2}/c\,. \quad (9)$$

Finally, we can obtain $t^{\mathrm{object}}_{\mathrm{H}_{mn}}$ representing the time when the object pulse emitted from D reaches $\mathrm{H_{mn}}$ as follows:

$$t^{\mathrm{object}}_{\mathrm{H_{mn}}} = t_{\mathrm{D}} + T_{\mathrm{D\to H_{mn}}}. \quad (10)$$

On the other hand, we consider $T_{\mathrm{H_y\to H_{mn}}}$ representing the time it takes the reference pulse propagates from $\mathrm{H_y}$ to $\mathrm{H_{mn}}$ on the hologram plate. $T_{\mathrm{H_y\to H_{mn}}}$ is given by

$$T_{\mathrm{H_y\to H_{mn}}} = x_{\mathrm{m}} \sin\theta_R / c\,. \quad (11)$$

Because the reference pulse reaches O at $t = 0$, we obtain the following equation by defining $t^{\mathrm{reference}}_{\mathrm{H_{mn}}}$ as the time when the reference pulse reaches $\mathrm{H_{mn}}$:

$$t^{\mathrm{reference}}_{\mathrm{H_{mn}}} = T_{\mathrm{H_y\to H_{mn}}}\,. \quad (12)$$

In LIF holography, interference fringes are formed by the object and reference pulses when the two pulses reach $\mathrm{H_{mn}}$ at the same time. Therefore, object pulses can be recorded at $\mathrm{H_{mn}}$ when the following equation is satisfied:

$$t^{\mathrm{object}}_{\mathrm{H_{mn}}} = t^{\mathrm{reference}}_{\mathrm{H_{mn}}}\,. \quad (13)$$

Practically, because an ultrashort light pulse has temporal width, we can replace Equation (13) with the following inequality:

$$\left| t^{\mathrm{object}}_{\mathrm{H_{mn}}} - t^{\mathrm{reference}}_{\mathrm{H_{mn}}} \right| < t_{\mathrm{pulse}}\,, \quad (14)$$

where t_{pulse} indicates the temporal width of an ultrashort light pulse. We can calculate CGHs of LIF holography by using aggregate of point-light sources satisfying Equation (14).

3.2 Proposed model with FDTD method

General CGH calculation doesn't require temporal information of light, whereas FDTD method treats temporal information of light. Then, we consider FDTD method is compatible with LIF holography. Although the whole calculation required for LIF holography should be performed by FDTD method ideally, we cannot directly calculate CGHs of LIF holography because FDTD method is a memory-hogging approach. Then, we partially introduce FDTD method into the numerical simulation model. For simplicity, we apply FDTD method only to simulation of light pulse propagation only on the diffuser plate in this work. This means that we use FDTD method before light pulses are scattered on the diffuser plate. After scattering, we simulate light pulse propagation based on the same model as the conventional one.

4 Results

Fig. 3 shows the schematic diagram of the brief situation on the diffuser plate. We assumed the total reflection in glass substrate as an object. For simplicity of calculation, we ignored the thick of the glass substrate in z direction. In other words, we examined 2D-FDTD simulation in xy plane. We set the resolution of the FDTD-simulation region as 32,768×8,192 pixels. A medium with the refractive index of 1.5 was positioned in the simulation region as glass substrate as shown in Fig. 3. We set an ultrashort light pulse whose temporal width was 100 femtosecond. It had a spatially and temporally Gaussian profile. The central wavelength of the pulse (represented by λ) was set as 532 nm. We introduced the pulse into the left-side edge of the medium at an angle of 30 degrees. The spatial steps along x and y directions (represented by Δx and Δy) were defined as $\lambda/20$. We defined the time interval (represented by Δt) as $\Delta t = c\Delta x = c\Delta y = c\lambda/20$. To prevent light pulses with reflecting the boundary of the simulation region, we adopted Mur's absorbing boundary condition with second-order approximation [11].

Fig. 4 shows examples of the intensity distributions in light pulse propagation obtained by FDTD simulation. We can observe the light pulse propagates and reflects

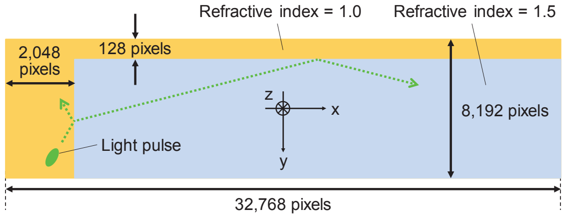


Fig. 3 Schematic diagram of brief situation on diffuser plate in proposed simulation model.

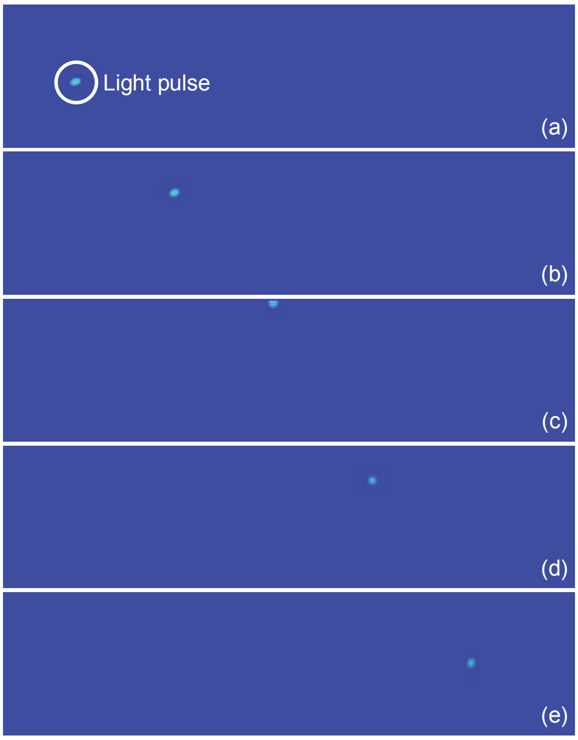


Fig. 4 examples of the intensity distributions in light pulse propagation obtained by FDTD simulation at time steps of (a) 10,000, (b) 23,000, (c) 36,000, (d) 49,000, and (e) 62,000, respectively.

in the boundary of the glass substrate. By using these results and the simulation model based on Fig. 4, we calculated a CGH of LIF holography by Equations (1) and (2). We set the pixel pitch and the resolution of a CGH as 2.0 μm and 65,536×2,048 pixels, respectively. We positioned the CGH at 100 mm away from the diffuser plate. Here, the actual size of the results of FDTD simulation was much smaller than that of the CGH because FDTD method is memory-hogging approach and we couldn't store sufficient data in memory at once. Then, we assumed a virtual magnification system to magnify the phenomenon of light pulse propagation [12]. The magnification rate was set as 100. The reference pulse was incident to the hologram plane at an angle of 45 degrees. We numerically reconstructed the calculated CGH by diffraction calculation [13]. Here, we consulted Ref. [7] to obtain moving pictures of LIF holography digitally. Fig. 5 shows five frames extracted from the obtained moving pictures. These results agreed well with the experimental result reported by Kubota and Awatsuji [8]. We successfully demonstrated numerical reconstruction of light pulse propagation with total reflection by LIF holography. We also succeeded in demonstrating the validity of the proposed simulation model for CGH calculation of LIF holography.

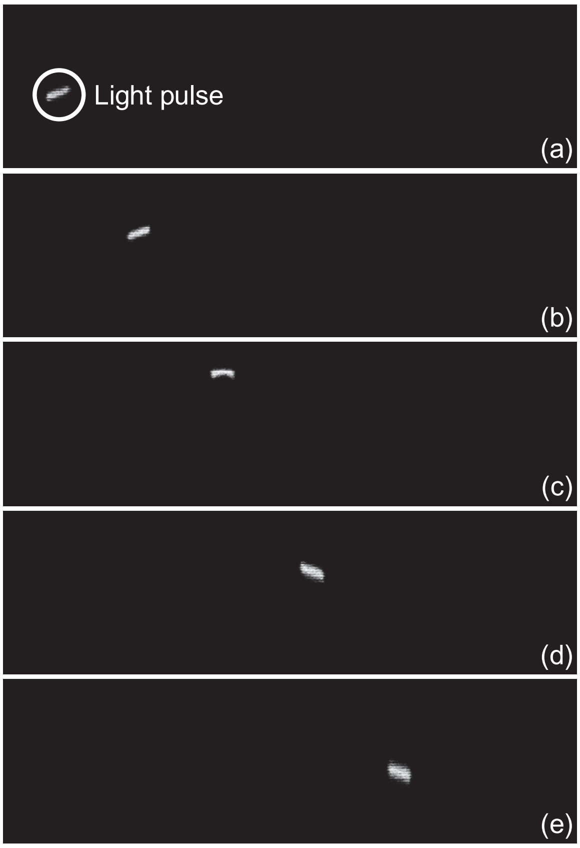


Fig. 5 Five frames extracted from moving pictures. Time passes from (a) to (e).

5 CONCLUSIONS

We proposed a novel numerical simulation model for LIF holography based on not only ray tracing but also FDTD method. By the proposed model, we calculated CGHs of LIF holography. Finally, we successfully demonstrated the reconstruction of light pulse propagation with total reflection, and the validity of the proposed simulation model for hologram calculation. This work will contribute to observation of nonlinearly ultrafast phenomena and elucidation of their dynamics and mechanism.

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REFERENCES

[1] M. Fujimoto, *et al.*, "Femtosecond time-resolved optical polarigraphy: imaging of the propagation dynamics of intense light in a medium," Opt. Lett., Vol. 24, pp. 850–852 (1999).

[2] R. C. Reddick, *et al.*, "New form of scanning optical microscopy," Phys. Rev. B, Vol. 39, pp. 767–770 (1989).

[3] A. Velten, *et al.*, "Femto-photography: capturing and visualizing the propagation of light," ACM TOG, Vol. 32, 44 (2013).

[4] G. Gariepy, *et al.*, "Single-photon sensitive light-in-flight imaging," Nat. Commun., Vol. 6, 6021 (2015).

[5] N. Abramson, "Light-in-flight recording by holography," Opt. Lett., Vol. 3, pp. 121–123 (1978).

[6] T. Kubota, *et al.*, "Moving picture recording and observation of three-dimensional image of femtosecond light pulse propagation," Opt. Express, Vol. 15, pp. 14348–14354 (2007).

[7] T. Kakue, *et al.*, "Digital light-in-flight recording by holography by use of a femtosecond pulsed laser," IEEE J. Sel. Top. Quantum Electron., Vol. 18, pp. 479–485 (2012).

[8] T. Kubota and Y. Awatsuji, "Observation of light propagation by holography with a picosecond pulsed laser," Opti. Lett., Vol. 27, pp. 815–817 (2002).

[9] K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," IEEE Trans. Antennas Propag. Vol. 14, pp. 302–307 (1966).

[10] T. Kakue, *et al.*, "Review of real-time reconstruction techniques for aerial-projection holographic displays," Opt. Eng. Vol. 57, 061621 (2018).

[11] G. Mur, "Absorbing Boundary Conditions for the Finite-Difference Approximation of the Time-Domain Electromagnetic-Field Equations," IEEE Trans. Electromagn. Compat., Vol. EMC-23, 377–382 (1981).

[12] T. Inoue, *et al.*, "Simultaneous recording of multiple and magnified motion pictures of polarized light propagation in three-dimensional medium by digital light-in-flight recording by holography," Proc. DHIP 2018, 19P01 (2018).

[13] J. W. Goodman, "Introduction to Fourier Optics," ROBERTS and COMPANY, Englewood (2004).