# Evaluation of Image Resolution of Aerial Image Based on Slanted Knife Edge Method

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### ABSTRACT

*We report image resolution measurement of an aerial image based on the slanted edge method. From the slanted edge image, edge spread function (ESF) is calculated by projecting the profile with some methods. We have compared three projecting method to obtain ESF. Furthermore, the proposed method is utilized for AIRR.*

## 1 INTRODUCTION

Aerial displays are expected to open a new market for digital signage, entertainment, and car equipment. Imaging with passive optics is a typical technique which forms a real image in the air. For example, an aerial image is formed by use of a dihedral corner reflector array (DCRA) [1], a crossed slit-mirror array [2], and a retro-reflector [3]. We have proposed a method of aerial imaging by retro-reflection (AIRR) [4].

Image resolution is one of the most important performance specifications of aerial display techniques. However, there is no standard method for evaluating the aerial image resolution. We have proposed a method to evaluate aerial image resolution, which employs contrast transfer function (CTF) [5]. However, there was a problem that several recorded images were required and fixed spatial frequency was determined at time of recorded image.

The purpose of this paper is to evaluate the image resolution of aerial image by use of the slanted knife edge. We have simulated the influence of the projection method calculating edge spread function (ESF). Furthermore, the modulation transfer function (MTF) curve of the aerial image formed using AIRR is measured using the slanted knife edge method.

## 2 PRINCIPLE

### 2.1 Aerial Imaging by Retro-Reflection (AIRR)

We have utilized a retro-reflector to form aerial display. The principle of AIRR is shown in Fig. 1. The setup consists of a light source, a beam splitter, and a retro-reflector. Light rays from the light source impinge on the beam splitter. Then, rays reflected by the beam splitter impinge on the retro-reflector. After retro-reflection, the rays reflected back along their incident directions. Rays that transmit through the beam splitter converge in the air. Thus, the aerial image of the light source is formed at the plane-symmetrical position of the light source regarding the beam splitter.

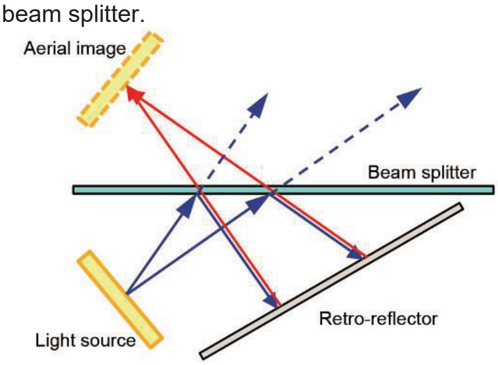



**Fig. 1 Principle of AIRR.**

### 2.2 Slanted knife edge method

A conceptual diagram of calculating MTF is shown in Fig. 2. The slanted knife edge method [6] estimates the MTF curve by calculating a region of interest (ROI) in a recorded edge image, which is the image of a slightly slanted knife edge illuminated by a uniform divergent light. The edge position in every data line of slanted edge image is corrected. All data is projected onto the projection axis and superimposed. The values of projected pixels collected in each bin are averaged. The obtained ESF $ESF(x')$ can effectively increase the sampling interval and eliminate the aliasing problem. The line spread function (LSF) $LSF(x')$ is obtained by

$$LSF(x') = \frac{d}{dx'} ESF(x') \,. \tag{1}$$

$MTF(x')$ is obtained by Fourier transform of LSF, is expressed by

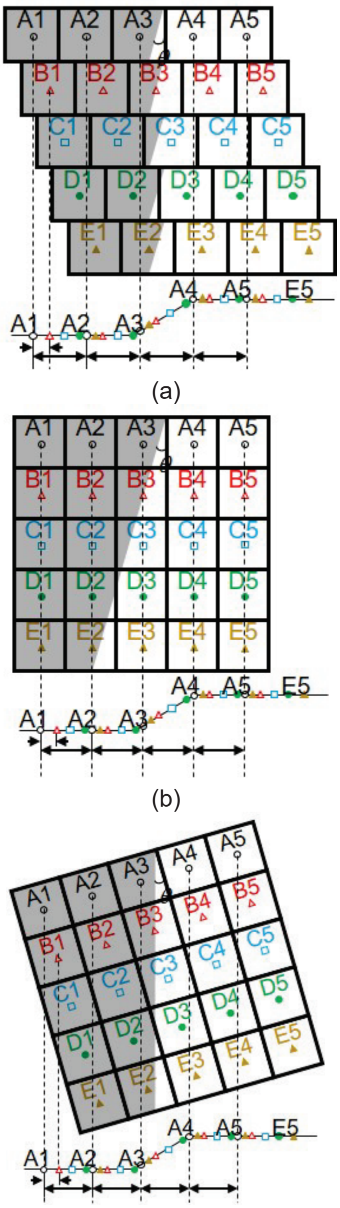
$$MTF(x') = |\mathcal{F}[LSP(x')]| \,. \tag{2}$$

Select ROI of slanted edge image.
↓
Obtain EST curve by projecting pixel data.
↓
Remove noise of ESF curve using wavelet denoise.
↓
Derive LSF curve with differential of ESF curve.
↓
Calculate MTF curve using Fourier transform.

**Fig. 2 Slanted edge method for calculating MTF.**

### 2.3 Projection method of edge spread function

In slanted edge method, it is difficult to obtain a stable MTF curve due to the noise is amplified by differentiating the ESF curve. We examine three types of projection methods calculating ESF curve, as shown in Fig. 3. In method A, shown in Fig. 3 (a), calculates the edge position of each data line and superimpose on the edge position. In method B, shown in Fig. 3 (b), the data in the edge image are projected on x-axis direction and superimpose. In method C, shown in Fig.3 (c), the data in the edge image are projected onto the vertical to the edge and superimpose.




(a) (b) (c)

**Fig. 3 Projection method of slanted edge image.**
(a) Method A is superimposition on the edge position,
(b) Method B is projection on x-axis direction, and
(c) Method C is projection on vertical to the edge.

## 3 NUMERICAL SIMULATIONS

### 3.1 Examination of simulated edge image with noise

In this simulation, we have investigated the influence in case of constructing ESF with three type of projection methods for slanted edge images with noise.

The blurred image $g(x, y)$ is obtained by

$$g(x,y) = f(x,y) * h(x,y) \tag{3}$$

where $f(x, y)$ is the ideal edge image, and $h(x, y)$ is the blur function.

The edge image with edge angle of 5 degrees that was obtained by convolving Gaussian blur according to Eq. (1) is shown in Fig. 4 (a). Fig. 4 (b) shows a noise image obtained by adding white noise with $10^{-4}$ noise variance to the edge image shown in Fig. 4 (a).

(a) (b)

**Fig. 4 Simulated 5-degrees-slanted knife edge images (a) without a white noise and (b) with a white noise.**

### 3.2 Simulation Result

In accordance with Sec. 2.2, we have simulated the sharpness of edge image. Fig. 5 shows the ESF curves with slanted edge images obtained by use of projection method as shown Fig. 3. Fig. 6 shows LSF curves of ESF curves shown in Fig. 5 which are calculated according to Eq. (1). MTF curves that were obtained according to Eq. (2) are shown in Fig. 7.

Table 1 shows the standard deviation of the errors between MTF curves with noise and without noise is calculated. In the case of the slanted edge image without a white noise, there is no significant difference in the MTF curves obtained with the three types of projection methods. The LSF curve calculated from the ESF curve using projection method A is strongly affected by noise. Among the three types of projection method, method C gives the lowest error, and method A gives the highest.

**Table 1 Standard deviation of the MTF errors by projection method.**

| | Method A | Method B | Method C |
|---|---|---|---|
| S.D. | 0.0530 | 0.0016 | 0.0007 |

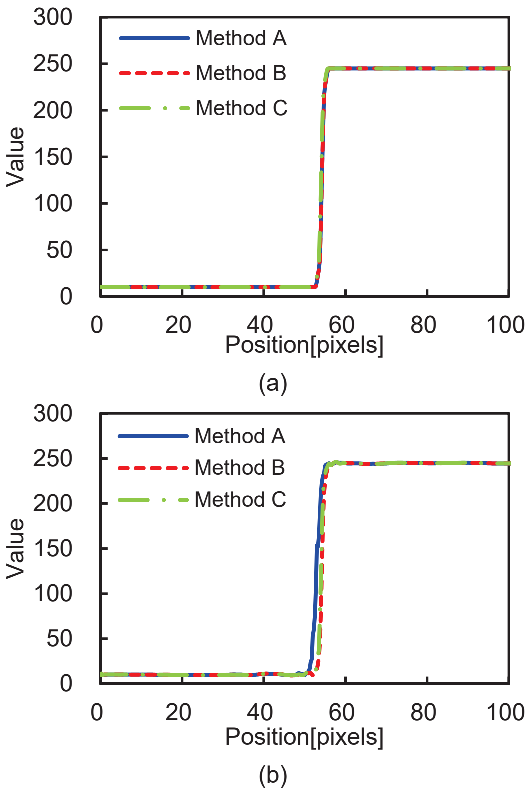



**Fig. 5 Calculated ESF curves using a slanted edge images (a) without noise and (b) with a noise.**

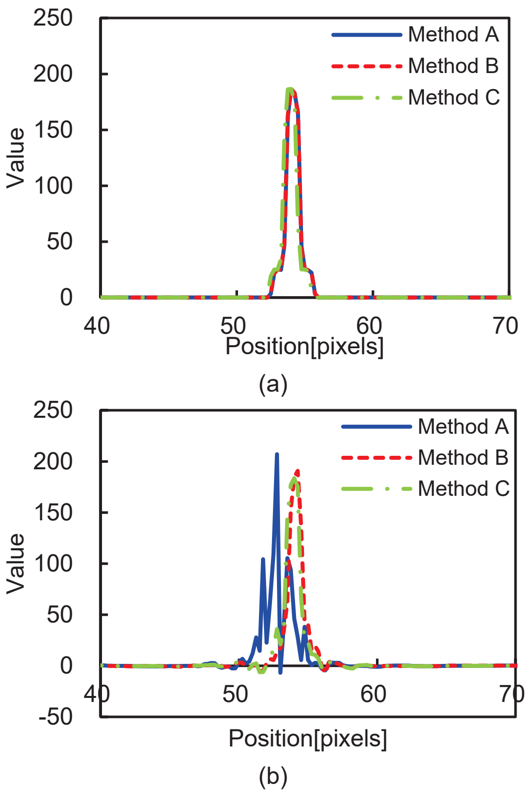



**Fig. 6 Calculated LSF curves using a slanted edge images (a) without noise and (b) with a noise.**

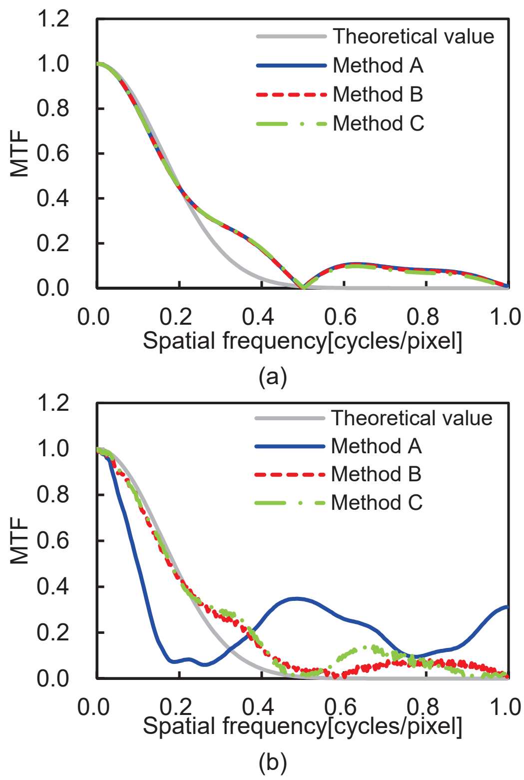



**Fig. 7 Obtained MTF curve from a slanted edge images (a) without noise and (b) with a noise.**

## 4 EXPERIMENTS

### 4.1 Experimental Setup

In this experiment, we investigate the influence of MTF curves of aerial image obtained using three projection methods. We measure the sharpness of aerial images formed by use of this retro-reflector, which is Nikkalite CRG. Fig. 8 shows the experimental setup to measure MTF of aerial imaging optics. The beam splitter is a half mirror (transmittance and reflectance are close to 50%). We use a digital camera (Nikon, D5500). The camera was rolled by 5 degrees. The recording conditions of the camera are ISO 400, 1/25 second exposure time, and F-number 4.5. Its 35mm equivalent focal length is 52 mm. The distance $a$ between a light source and a beam splitter is 130 mm. The distance $b$ between the aerial image and the camera is 265 mm.

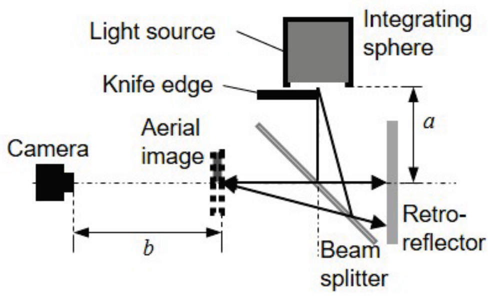



**Fig. 8 Experimental setup to obtain the aerial image of a slanted knife edge.**

### 4.2 Experimental Result

An aerial image of the slanted knife edge formed with AIRR is shown in Fig. 9 (a). The ROI image indicated by the red frame in Fig. 9 (a) is shown in Fig. 9 (b). Fig. 10 shows the ESF curve obtained by projecting and binning the aerial edge image selected by ROI. Fig. 11 shows LSF curves of the ESF curves which are calculated according to Eq. (1). The MTF curves that were obtained according to Eq. (2) are shown in Fig. 12. Method A is affected by noise. The MTF curves obtained by LSF used projection method B and C showed no significant difference. There was no significant influence of the side lobe.

## 5 CONCLUSION

We have measured MTF of the aerial imaging optics based on the slanted edge method. The influence of three kinds of projection methods calculating edge spread function were investigated. Method C, which projects the profile vertically to the edge showed the best performance among the three methods.

## REFERENCES


[1] S. Maekawa, K. Nitta, and O. Matoba, “Transmissive Optical Imaging Device with Micromirror Array,” Proc. SPIE 6392, 63920E (2006).

[2] ASKA3D, https://aska3d.com/.

[3] C. B. Burckhardt, R. J. Colloer, and E. T. Doherty, “Formation and inversion of pseudoscopic Images,” Appl. Opt. **7** (4), 627-631 (1968).

[4] H. Yamamoto, Y. Tomiyama, and S. Suyama, “Floating aerial LED signage based on aerial imaging by retro-reflection (AIRR),” Opt. Express **22** (22), 26919-26924 (2014).

[5] N. Kawagishi, K. Onuki, and H. Yamamoto, “Comparison of divergence angle of retro-reflectors and sharpness with aerial imaging by retro-reflection (AIRR),” IEICE Trans. Electron, **E100-C** (11), 958-964 (2017).

[6] ISO 12233: 2014, “Photography - Electronic still picture imaging - Resolution and spatial frequency response”.

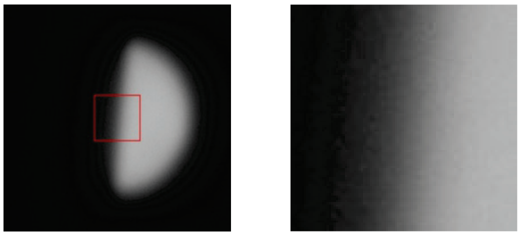




**Fig. 9 Slanted edge image of aerial image.**
(a) Recorded image of slanted edge. (b) ROI for estimating MTF.

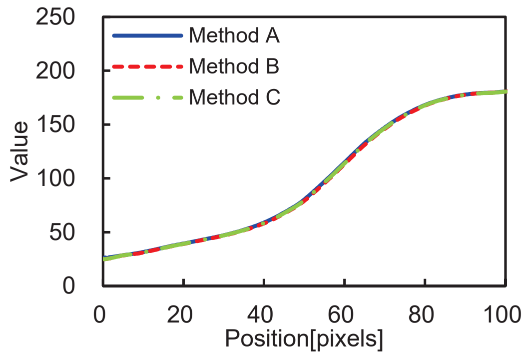



**Fig. 10 ESF curves of aerial image formed by use of AIRR calculating slanted edge method.**

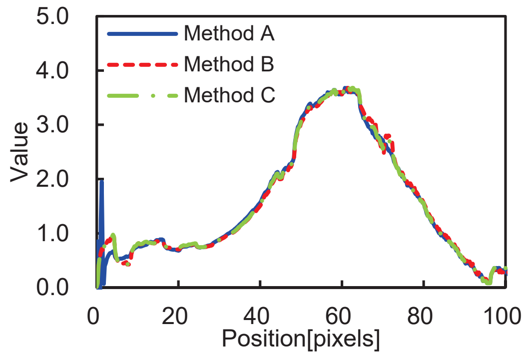



**Fig. 11 LSF curves of aerial image formed by use of AIRR calculating slanted edge method.**

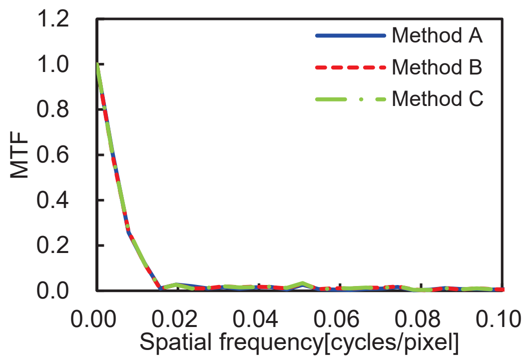



**Fig. 12 MTF curves of aerial image formed by use of AIRR calculating slanted edge method.**