Mathematical mechanism of an inverse problem for tephra transport and dispersal model

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Estimation of tephra mass released along the eruption column is crucial for the characterization of explosive volcanic eruptions systems and for the volcanic hazard mitigations. For example, the uncertainties of assumed tephra mass distribution as the source term for tephra transport and dispersal models critically influence the accuracy of the tephra forecast. In addition, the estimated tephra mass distribution can be used for verification and comprehension of some eruption column models which have been developed in previous studies (e.g. Woods & Bursik, 1991) .

We are studying an inverse problem of a model which relates the tephra mass distribution along the eruption column to the tephra fallout deposits. The model is expressed as a linear system which builds on the advection-fall-dispersion-sedimentation equation. The inverse problem is, therefore reduced to a ‘linear inverse problem’. In this study, we investigated the properties of this linear inverse problem on the basis of the singular value decomposition (SVD) using numerical experiments that generate pseudo-observations.

The SVD analysis indicates that present inverse problem is that of an ‘ill-conditioned’ system, where most of singular values are nearly zero. This characteristic comes from the fact that tephra particles released from high altitudes tend to lose the information of their sources because of diffusion. The small singular values cause an unstable and non-physical inverse solution to be completely dominated by small amount of observation error (referred to as over-fitting). Therefore, the present inverse problem requires careful consideration to obtain stable and meaningful solutions. In our study, the regularization and the optimization of the vertical grid are applied to produce such solutions. In the regularization, we do not include singular vectors with small singular value as the basis function expressing the inverse solution, such that the solution can be less sensitive to observation error. In the optimization of the vertical grids, we reduce the number of vertical grids of the model parameters using linear interpolation to avoid singular vectors with small singular values. Combining these two methods, we successfully find approximate solutions without over-fitting. In the presentation, we also discuss how the eruption conditions (e.g., magma discharge rate) are constrained by the model parameters estimated from the present method.

Keywords: inverse problem, singurar value, singurar value decomposition, eruption column, tephra fallout deposit, volcanic eruption

