# Designing high-order finite difference scheme for magnetohydrodynamics: shock capturing and divergence-free conditions.

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Space and astrophysical plasmas are rich in dynamic phenomena such as convection, eruption, shock, accretion, and so on. Their macroscopic dynamics is well described by magnetohydrodynamics (MHD). Since the system of MHD equations are highly nonlinear, a numerical simulation is an indispensable tool to reveal its complicated physics.
The plasma is frequently associated with supersonic flows such as the coronal mass ejection, supernova, and jet, which yield various shocks and discontinuities. Furthermore, these flows are almost inviscid, thus will become turbulent. Reliable MHD simulations need to resolve these phenomena simultaneously, which is a contradictory issue for computational fluid dynamics and thus is challenging. Moreover, the MHD simulation should carefully handle errors of the divergence for magnetic field, which is not necessarily free in numerical simulations.
Many works have been devoted to develop exact or approximate Riemann solvers (upwind schemes) for (M)HD to accurately capture shocks and discontinuities. Nowadays, such shock capturing schemes are adopted as a standard method for MHD simulations (Kritsuk et al. 2011). On the other hand, various high-order interpolation techniques are proposed to improve the resolution of small scale structure (e.g., turbulence), and they can be incorporated into shock capturing schemes.
Shock capturing schemes are based on the finite volume method, which automatically satisfies the conservation laws but has a difficulty in achieving high order of accuracy in multi-dimension. The finite difference method is rather convenient for designing multidimensional high-order scheme. Conservative finite difference schemes have been proposed by approximating fluxes to high order, and succeeded in high resolution MHD simulations (Jiang et al. 1999; Mignone et al. 2010).
We consider another type of the conservative finite difference scheme for MHD, which interpolates physical variables to high order and utilize a variety of Riemann solvers to capture shocks. We also take special care of the divergence-free condition for magnetic field. Combination of the upwind scheme and the constrained transport (UCT) method, which satisfies divergence-free condition within machine accuracy without violating upwind property (via Riemann solvers), is thought to be a powerful strategy especially for low beta plasmas. We test various type of the UCT method. In this paper, we will present details of our code design and its performance, especially focusing on the comparison among different interpolation techniques, Riemann solvers, and UCT methods.

Keywords: MHD simulation, Numerical method

 