Point and Line Attractors Emerging in the System Including the Interaction among Heat, Fluid Pressure and Dilatancy

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We show two geometrically different attractors, point and line attractors, emerge within the framework including the interaction among heat, fluid pressure and dilatancy associated with dynamic earthquake source process. To show that, we consider qualitative behavior of the solution orbit in the \phi-v space, where \phi and v describe the inelastic porosity and the slip velocity, respectively. We first consider nullclines, which are obtained by the conditions \dot{v}=0 and \dot{\phi}=0. For \dot{v}=0, the straight line v=0 and the curve v=1-\beta g(\phi) are nullclines, where g(\phi) is the function describing the porosity evolution law and \beta is a positive constant number. The curve v=1-\beta g(\phi)) on the \phi-v space will be referred to as C^{crit} henceforth. For \dot{\phi}=0, the straight line v=0 and the curve \phi=1 are found to be nullclines. Clearly, the line v=0 is the common nullcline for both equations.

We simply assume that C^{crit} crosses the \phi-axis once, and C^{crit} is ascending with increasing \phi. We consider a solution orbit crossing the v-axis in the region $0<v_0<1$, where v_0 is the slip velocity at \phi=0, since v and \phi are normalized and take values between zero and unity. It should be noted that the orbit is not horizontal nor vertical at the point crossing the line v=0, even though the line is a nullcline. This occurs because v=0 is a nullcline for both equations; both the relationships \dot{v}=\dot{\phi}=0 are satisfied on the line v=0, which enables dv/d \phi to be nonzero there. Moreover, we can confirm the orbit connects the points (0, v_0) and (1, 1).

The solution orbits and the moving direction of the solution conclude that we have attractor and repeller on the \phi-axis; {(\phi_a, 0) | 0 \le \phi_a \le \phi_c} is an attractor, while {(\phi_r, 0) | \phi_c \le \phi_r \le 1} is a repeller, where \phi_a and \phi_r are the real numbers satisfying \phi_a < \phi_c and \phi_r > \phi_c, and \phi_c is the \phi value of the point where C^{crit} and \phi-axis cross. In particular, note that \phi_a and \phi_r take continuous values. These non-isolated fixed points appear because the line v=0 is a nullcline for both equations, and this is characteristic behavior of the present system. In addition, the point (1, 1) is also the attractor because C^{crit} and all orbits are absorbed into the point (1, 1). We can therefore summarize that the attractors are categorized into two geometrically different groups: they are given by the line {(\phi_a, 0) | 0 \le \phi_a \le \phi_c} or the point (1,1). The detail of g(\phi) does not affect the emergence of the attractors.

Keywords: heat, fluid pressure, dilatancy, solution orbit, attractor, repeller